

4. COMPETITIVE MARKETS

We studied how individual consumers and firms behave in Part I of the book. In Part II of the book, we studied how individual economic agents make decisions when there are strategic independence. What we have not considered carefully so far are the institutions under which economic agents make their decisions. This is the task that we are going to take up now. When the two basic types of economic agents, consumers and firms, make decisions, they interact through the market. Among the fundamental questions we want to answer are: How are prices and output determined under various market structures? And what are the welfare properties of these markets? We shall address these questions starting from a benchmark: the competitive economy, where there is a market for each good in the economy, there is perfect information, and each individual consumer and firm acts as a price taker.

4.1 Pareto Optimality and Competitive Equilibrium

Consider an economy consisting of I consumers (indexed by $i = 1, \dots, I$), J firms (indexed by $j = 1, \dots, J$), and L goods (indexed by $l = 1, \dots, L$). Consumer i 's utility from consuming bundle $x_i = (x_{1i}, \dots, x_{Li})$ is $u_i(\cdot)$, where $x_i \in X_i \subset R^L$, and X_i is called i 's consumption set. The initial endowment of good l is $\omega_l \geq 0$ for all $l = 1, \dots, L$. Each firm j 's production possibilities are represented by its production set $Y_j \subset R^L$, and $y_j = (y_{1j}, \dots, y_{Lj}) \in Y_j \subset R^L$. The production vectors of the J firms are $(y_1, \dots, y_J) \in R^{LJ}$. The total amount of good l available in the economy is $\omega_l + \sum_j y_{lj}$.

Definition 1 *An economic allocation $(x_1, \dots, x_I, y_1, \dots, y_J)$ is a specification of a consumption vector for each consumer and a production vector for each firm in the economy. The allocation $(x_1, \dots, x_I, y_1, \dots, y_J)$ is feasible if*

$$\sum_i x_{li} \leq \omega_l + \sum_j y_{lj} \text{ for } l = 1, \dots, L.$$

We now suggest a concept about what constitutes an efficient allocation.

Definition 2 *A feasible allocation $(x_1, \dots, x_I, y_1, \dots, y_J)$ is Pareto optimal (or Pareto efficient) if there is no other feasible allocation $(x'_1, \dots, x'_I, y'_1, \dots, y'_J)$ such that $u_i(x'_i) \geq u_i(x_i)$ for all $i = 1, \dots, I$ and $u_k(x'_k) > u_k(x_k)$ for some k .*

In other words, an allocation is Pareto optimal if there is no other way to organize production and consumption in the economy that will make some one better off without making somebody else worse off. In a labor dispute, for instance, it would not be Pareto efficient if workers have a strike. While Pareto optimality seems a necessary requirement for any desirable allocation, it is usually not sufficient. There could be several Pareto optimal allocations, and the choice may depend on considerations largely out of economics, such as consideration of equality or fairness.

There is a slightly different concept of Pareto optimality that is also used, called weak Pareto optimality. A feasible allocation $(x_1, \dots, x_I, y_1, \dots, y_J)$ is weakly Pareto optimal if there is no other feasible allocation $(x'_1, \dots, x'_I, y'_1, \dots, y'_J)$ such that $u_i(x'_i) > u_i(x_i)$ for all $i = 1, \dots, I$. Clearly, a Pareto optimal allocation is weakly Pareto optimal.

On the other hand, if consumer preference is continuous and strongly monotonic, then a weakly Pareto optimal allocation is also Pareto optimal if the allocation is in the interior of each consumer's consumption set. A sketch of the proof is the following:

First notice that $u_i(\cdot)$ will be continuous and strictly increasing if preference is continuous and strongly monotonic. Now if a feasible allocation $(x_1, \dots, x_I, y_1, \dots, y_J)$ is weakly Pareto optimal but not Pareto optimal, then there exists some other feasible allocation $(x'_1, \dots, x'_I, y'_1, \dots, y'_J)$ such that $u_i(x'_i) \geq u_i(x_i)$ for all $i = 1, \dots, I$ and $u_k(x'_k) > u_k(x_k)$ for some k . Let $x''_{1k} = x'_{1k} - \epsilon$, and $x''_{lk} = x'_{lk}$ for $l = 2, \dots, I$; $x''_{1i} = x'_{1i} + \frac{\epsilon}{I}$, and

$x''_{li} = x'_{li}$ for all $i \neq k$ and all $l = 1, \dots, L$. Then for sufficiently small $\epsilon > 0$, we have

$$u_i(x''_{1i}, \dots, x''_{Li}) > u_i(x_{1i}, \dots, x_{Li}) \text{ for all } i = 1, \dots, I, \text{ a contradiction.}$$

We now define a competitive equilibrium. Suppose consumer i initially owns ω_{li} amount of good l , and $\sum_i \omega_{li} = \omega_l$. Consumer i 's endowments are $\omega_i = (\omega_{1i}, \dots, \omega_{Li})$. i also owns θ_{ij} shares of firm j 's profits, where $\sum_i \theta_{ij} = 1$.

Definition 3 *The allocation $(x_1^*, \dots, x_I^*, y_1^*, \dots, y_J^*)$ and price vector $p^* \in R^L$ constitute a competitive (or Walrasian) equilibrium if the following conditions are satisfied:*

(i) Profit maximization: For each j , y_j^* solves

$$\max_{y_j \in Y_j} p^* \cdot y_j.$$

(ii) Utility maximization. For each i , x_i^* solves

$$\begin{aligned} & \max_{x_i \in X_i} u_i(x_i) \\ \text{s.t. } & p^* \cdot x_i \leq p^* \cdot \omega_i + \sum_j \theta_{ij}(p^* \cdot y_j^*). \end{aligned}$$

(iii) Market clearing. For each good $l = 1, \dots, L$,

$$\sum_i x_{li}^* = \omega_l + \sum_j y_{lj}^*.$$

Lemma 1 *If the allocation $(x_1, \dots, x_I, y_1, \dots, y_J)$ and price vector $p \gg 0$ satisfy the market clearing conditions for all goods $l \neq k$, and if $p \cdot x_i = p \cdot \omega_i + \sum_j \theta_{ij}(p \cdot y_j)$ for all i , then the market for k also clears.*

Proof. Adding up the two sides of budget equations for all consumers, we have

$$\sum_i \sum_l p_l x_{li} = \sum_i \sum_l p_l \omega_{li} + \sum_i \sum_j \theta_{ij}(p \cdot y_j),$$

or

$$\sum_l \sum_i p_l x_{li} = \sum_l \sum_i p_l \omega_{li} + \sum_l \sum_i \sum_j \theta_{ij} (p_l y_{lj}),$$

or

$$\sum_l \sum_i (p_l x_{li} - p_l \omega_{li} - \sum_j \theta_{ij} p_l y_{lj}) = 0,$$

or

$$\sum_l p_l (\sum_i x_{li} - \omega_l - \sum_j y_{lj}) = 0,$$

or

$$\sum_{l \neq k} p_l (\sum_i x_{li} - \omega_l - \sum_j y_{lj}) = -p_k (\sum_i x_{ki} - \omega_k - \sum_j y_{kj}).$$

Since markets clear for all $l \neq k$, we have $\sum_i x_{li} - \omega_l - \sum_j y_{lj} = 0$ for all $l \neq k$. This, together with $p_k > 0$, implies $\sum_i x_{ki} - \omega_k - \sum_j y_{kj} = 0$. Thus the market for good k also clears. ■

4.2 Partial Equilibrium Competitive Analysis

We now undertake our analysis using the partial equilibrium approach. In this approach, we assume that the good (market) under analysis represents only a small part of the economy and a consumer's expenditure on it is only a small portion of the consumer's total expenditures. Thus, we can ignore the effects of this market on the prices on other markets, and we may also ignore the wealth effects on this good. We can therefore treat all other goods in the economy as a single composite good, called the numeraire, and normalize the price of the numeraire to 1.

Now consider an economy with only two goods, good l and the numeraire m (we will call this the two-good quasilinear model). Denote consumer i 's consumption of good l and the numeraire by x_i and m_i . Assume that consumer i 's utility function takes the quasilinear form:

$$u_i(m_i, x_i) = m_i + \phi_i(x_i).$$

Also assume that each consumer's consumption set is $R \times R_+$. Notice that we allow m_i to be negative. $\phi_i(x_i)$ is assumed to be bounded above and twice differentiable, with $\phi_i'(x_i) > 0$ and $\phi_i''(x_i) < 0$ for all $x_i \geq 0$. We normalize $\phi_i(0) = 0$. Let the price of good l be p .

There are J firms in the economy, each can produce $q_j \geq 0$ units of l using $c_j(q_j)$ amount of m . Thus $c_j(q_j)$ is j 's cost function. Assume $c_j'(q_j) > 0$ and $c_j''(q_j) \geq 0$ at all $q_j \geq 0$.

Each consumer i has a initial endowment of m , ω_{mi} , and $\sum_i \omega_{mi} = \omega_m$. There is no initial endowment for good l .

To find a competitive equilibrium of this economy, we proceed as follows:

For firms: given p^* , firm j 's output solves

$$\max_{q_j \geq 0} p^* q_j - c_j(q_j),$$

which has the necessary and sufficient f.o.c.

$$p^* \leq c_j'(q_j^*), \text{ with equality if } q_j^* > 0.$$

For consumers: given p^* , ω_{mi} , θ_{ij} , and q_j^* , consumer i 's consumption bundle (m_i^*, x_i^*) solves

$$\begin{aligned} & \max_{m_i \in R, x_i \in R_+} m_i + \phi_i(x_i) \\ \text{s.t. } m_i + p^* x_i &= \omega_{mi} + \sum_j \theta_{ij} (p^* q_j^* - c_j(q_j^*)). \end{aligned}$$

Or, equivalently, (m_i^*, x_i^*) solves

$$\max_{m_i \in R, x_i \in R_+} \phi_i(x_i) - p^* x_i + \omega_{mi} + \sum_j \theta_{ij} (p^* q_j^* - c_j(q_j^*)),$$

which has the necessary and sufficient f.o.c.

$$\phi_i'(x_i^*) \leq p^*, \text{ with equality if } x_i^* > 0.$$

For market clearing: We shall adopt the convention of identifying the equilibrium allocation by $(x_i^*, \dots, x_I^*, q_1^*, \dots, q_J^*)$, with the understanding that $m_i^* = \omega_{mi} + \sum_j \theta_{ij}(p^* q_j^* - c_j(q_j^*)) - p^* x_i^*$ and firm j 's equilibrium usage of m as an input is $c_j(q_j^*)$. From the lemma earlier, both markets will clear if and only if the market for l clears, that is,

$$\sum_i x_i^* = \sum_j q_j^*.$$

Thus, allocation $(x_i^*, \dots, x_I^*, q_1^*, \dots, q_J^*)$ and price p^* constitute a competitive equilibrium if the three conditions below are satisfied:

$$p^* \leq c'_j(q_j^*), \text{ with equality if } q_j^* > 0, \text{ for all } j = 1, \dots, J.$$

$$\phi'_i(x_i^*) \leq p^*, \text{ with equality if } x_i^* > 0, \text{ for all } i = 1, \dots, I.$$

$$\sum_i x_i^* = \sum_j q_j^*.$$

For any interior solution, these conditions say that in equilibrium, price equals marginal cost for each firm, price equals marginal utility for each consumer, and aggregate demand and supply for good l must be equal. Also notice that the equilibrium allocation and price are independent of the distribution of endowments and ownership shares. This is due to the assumption that m_i can be negative, as well as to the assumption of quasilinear utility function.

The competitive equilibrium of this model can also be found using the traditional demand and supply analysis. Assume that $\text{Max}_i \phi'_i(0) > \text{Min}_j c'_j(0)$, which guarantees $x^* > 0$.

Given any $p > 0$, we can find a unique x_i such that $\phi'_i(x_i) \leq p$, with equality if $x_i > 0$, for all $i = 1, \dots, I$. Thus consumer i 's demand for good l is $x_i(p)$. $x_i(p) > 0$ iff $\phi'_i(0) > p$. $x_i(p)$ is continuous and nonincreasing in p , and is strictly decreasing in p when $\phi'_i(0) > p$. The aggregate demand function for good l is $x(p) = \sum_i x_i(p)$, which is continuous and nonincreasing in p , and is strictly decreasing in p when $p < \text{Max}_i \phi'_i(0)$.

Similarly, for any $p > 0$, and assume $c_j''(q_j) > 0$, we can find a unique q_j from the firm j 's profit maximization condition. Firm j 's supply function is therefore $q_j(p)$. We have $q_j(p) > 0$ iff $c_j'(0) < p$. $q_j(p)$ is continuous and nondecreasing in p , and is strictly increasing in p if $c_j'(0) < p$. The aggregate supply function for good l is $q(p) = \sum_j q_j(p)$, which is continuous and nondecreasing in p , and is strictly increasing in p when $p > \text{Min}_j c_j'(0)$.

The equilibrium price is found where aggregate demand equals aggregate supply, or $x(p^*) = q(p^*)$. It is easy to verify that under our assumptions, p^* exists uniquely. What happens if $c_j''(q_j) = 0$?

When market conditions change, market outcomes change as well. To see how market outcomes change in response to changes in market conditions, the analysis is known as comparative statics analysis.

Example 1 *The effects of a sales tax. Suppose that under a new sales tax consumers must pay $t > 0$ for each unit of good l consumed. (a) Determine the new market price after the tax. (b) How will the unit cost of the good to consumers and the unit revenue received by firms be affected with a marginal change in the tax rate? (c) For the same t , does it matter to consumers whether the tax is paid by the consumers or firms?*

Answer: (a) Let the aggregate demand function be $x(p)$, and the aggregate supply function be $q(p)$. Notice that at price p and tax t , the aggregate demand is $x(p + t)$ and the aggregate supply is $q(p)$. Therefore the equilibrium market price $p^*(t)$ solves

$$x(p^*(t) + t) = q(p^*(t)).$$

(b) Assuming $x(\cdot)$ and $q(\cdot)$ are differentiable, we have

$$x'(p^*(t) + t)(p^{*'}(t) + 1) = q'(p^*(t))p^{*'}(t).$$

That is

$$p^{*'}(t) = \frac{x'(p^*(t) + t)}{q'(p^*(t)) - x'(p^*(t) + t)}.$$

Since $x'(p^*(t) + t) < 0$, and $q'(p^*(t)) \geq 0$, we have $0 \geq p^*(t) \geq -1$. Thus, except when $q'(p^*(t)) = 0$ or $q'(p^*(t)) = \infty$, a marginal increase in t reduces the unit revenue received by firms and increases the unit cost to consumers, but each in a less magnitude than the tax increase. When $q'(p^*(t)) = 0$, which means that the curve of $q(p)$ is vertical, the impact of the tax is born entirely by firms. When $q'(p^*(t)) = \infty$, which means that the curve of $q(p)$ is horizontal, the impact of the tax is born entirely by consumers.

(c) In this case, the equilibrium market price $p_f^*(t)$ solves

$$x(p_f^*(t)) = q(p_f^*(t) - t).$$

But then $p_f^*(t)$ must equal $p^*(t) + t$, since $p^*(t)$ solves $x(p^*(t) + t) = q(p^*(t))$. Therefore it does not matter to consumers whether consumers or firms pay the tax, since the real unit cost of the good to the consumers will be the same in both cases.

Example 2 Suppose that J firms produce good l , each with a differentiable cost function $c(q, \alpha)$ that is strictly convex in q , where α is an exogenous parameter and $\partial c(q, \alpha) / \partial \alpha > 0$. The aggregate demand function for good l is $x(p)$, with $x' \leq 0$. Find the marginal change in a firm's profits with respect to α in equilibrium.

Let the equilibrium output of each firm be $q(\alpha)$ and the equilibrium price be $p(\alpha)$. The profit of each firm is, given $p(\alpha)$,

$$\pi(q, \alpha) = qp(\alpha) - c(q, \alpha).$$

The equilibrium $q(\alpha)$ and $p(\alpha)$ are obtained by solving

$$p(\alpha) = c_q(q(\alpha), \alpha),$$

$$x(p(\alpha)) = Jq(\alpha).$$

And since

$$q'(\alpha) = \frac{1}{J} x'(p(\alpha)) p'(\alpha),$$

we have

$$p'(\alpha) = c_{qq}(q(\alpha), \alpha) \frac{1}{J} x'(p(\alpha)) p'(\alpha) + c_{q\alpha}(q(\alpha), \alpha),$$

or

$$p'(\alpha) = \frac{c_{q\alpha}(q(\alpha), \alpha)}{1 - c_{qq}(q(\alpha), \alpha) \frac{1}{J} x'(p(\alpha))}.$$

Now, the equilibrium profit of each firm is

$$\pi(q(\alpha), \alpha) = q(\alpha)p(\alpha) - c(q(\alpha), \alpha).$$

Using the Envelop Theorem, we have

$$\begin{aligned} \frac{d\pi(q(\alpha), \alpha)}{d\alpha} &= \frac{\partial\pi(q(\alpha), \alpha)}{\partial\alpha} = q(\alpha)p'(\alpha) - c\alpha(q(\alpha), \alpha) \\ &= \frac{q(\alpha)c_{q\alpha}(q(\alpha), \alpha)}{1 - c_{qq}(q(\alpha), \alpha) \frac{1}{J} x'(p(\alpha))} - c\alpha(q(\alpha), \alpha). \end{aligned}$$

The fundamental Welfare Theorems in a Partial Equilibrium Context

We now investigate the relations between Pareto optimal allocations and competitive equilibria in the partial equilibrium model developed earlier.

Suppose at some allocation the consumption and production of good l is $(x_1, \dots, x_I, q_1, \dots, q_J)$. Then the total amount of numeraire that is available for distribution among consumers is $\omega_m - \sum_j c_j(q_j)$. Because the utility function is quasilinear, there can be unlimited unit-for-unit transfer of utility across consumers through transfers of the numeraire. The set of utilities that can be attained for the I consumers by appropriately distributing the available amount of the numeraire is

$$\left\{ (u_1, \dots, u_I) : \sum_{i=1}^I u_i \leq \sum_{i=1}^I \phi_i(x_i) + \omega_m - \sum_{j=1}^J c_j(q_j) \right\}.$$

Suppose $(x_1^*, \dots, x_I^*, q_1^*, \dots, q_J^*)$ solves

$$\begin{aligned} & \max_{(x_1, \dots, x_I) \geq 0, (q_1, \dots, q_J) \geq 0} \sum_{i=1}^I \phi_i(x_i) - \sum_{j=1}^J c_j(q_j) + \omega_m \\ \text{s.t. } & \sum_i x_i = \sum_j q_j. \end{aligned}$$

Then any allocation with $(x_1^*, \dots, x_I^*, q_1^*, \dots, q_J^*)$ must be Pareto optimal, and those Pareto optimal allocations can differ only in the distribution of the numeraire among consumers. The value of the term $\sum_{i=1}^I \phi_i(x_i) - \sum_{j=1}^J c_j(q_j)$ is called aggregate surplus. Thus optimal consumption and production maximizes aggregate surplus subject to the market clearing condition. The necessary and sufficient f.o.c. for the maximization problem are:

$$\phi'_i(x_i^*) \leq \mu, \text{ with equality if } x_i^* > 0, \text{ for } i = 1, \dots, I,$$

$$\mu \leq c'_j(q_j^*), \text{ with equality if } q_j^* > 0, \text{ for } j = 1, \dots, J,$$

$$\sum_i x_i^* = \sum_j q_j^*.$$

If we replace μ by p^* , these are the exactly same conditions that characterize the competitive equilibria. We have therefore shown:

Proposition 1 (*The first Fundamental Theorem of Welfare Economics*) *If the price p^* and allocation $(x_1^*, \dots, x_I^*, q_1^*, \dots, q_J^*)$ constitute a competitive equilibrium, then this allocation is Pareto optimal.*

We can also develop a converse to the result above. Recall that in a competitive equilibrium of our quasilinear model, p^* , $(x_1^*, \dots, x_I^*, q_1^*, \dots, q_J^*)$, and profits of firms are all independent of consumer's wealth. Thus by properly transferring the endowment of the numeraire among consumers, we can achieve any utility vector along the boundary of the utility possibility set: $\{(u_1, \dots, u_I) : \sum_{i=1}^I u_i = \sum_{i=1}^I \phi_i(x_i^*) + \omega_m - \sum_{j=1}^J c_j(q_j^*)\}$. We therefore have:

Proposition 2 (*The second Fundamental Theorem of Welfare Economics*) *For any Pareto optimal levels of utility (u_1^*, \dots, u_I^*) , there are transfers of the numeraire commodity (T_1, \dots, T_I) satisfying $\sum_i T_i = 0$, such that a competitive equilibrium reached from the endowments $(\omega_{m1} + T_1, \dots, \omega_{mI} + T_I)$ yields precisely the utilities (u_1^*, \dots, u_I^*) .*

Since the competitive price is equal to the shadow price on the resource constraint for good l in the Pareto optimal problem, we can think the price in a competitive equilibrium reflects precisely its marginal social value. In a competitive equilibrium, for each firm, the marginal cost of production equals marginal social value, and, for each consumer, the marginal benefit of consumption of the product is equal to the marginal cost of producing the product.

An alternative way to find the set of Pareto optimal allocation is to maximize one consumer's utility subject to the condition that other consumers' utility levels are not below some fixed levels, and to other resource and technological constraints.

Welfare Analysis in the (Quasilinear) Partial Equilibrium Model

Suppose that there is some social welfare function $W(u_1, \dots, u_I)$ that assigns a social welfare value to every utility vector (u_1, \dots, u_I) . Given the consumption and production of good l as $(x_1, \dots, x_I, q_1, \dots, q_J)$, the set of all achievable utility vectors is

$$\left\{ (u_1, \dots, u_I) : \sum_{i=1}^I u_i \leq \sum_{i=1}^I \phi_i(x_i) - \sum_{j=1}^J c_j(q_j) + \omega_m \right\}.$$

We can think of maximizing social welfare as involving two steps: One is to maximize the (Marshallian) aggregate surplus

$$S(x_1, \dots, x_I, q_1, \dots, q_J) = \sum_{i=1}^I \phi_i(x_i) - \sum_{j=1}^J c_j(q_j).$$

And the other is then to distribute ω_m across the consumers to maximize the social welfare function. This can be illustrated in a diagram when $I = 2$.

Therefore, assuming that the numeraire can be properly transferred, maximizing social welfare (for any social welfare function) is equivalent to maximizing aggregate surplus, and changes in social welfare can be measured by changes in aggregate surplus (also called social surplus). In many situations, the social surplus can in turn be measured in terms of the area between the aggregate demand and supply functions for good l . This requires two assumptions: (i) Denote the total consumption of good l as $x = \sum_i x_i$. We assume that the allocation x is optimal across the consumers. That is, $\phi'_i(x_i) = P(x)$ for all i . This is assured if all consumers are price takers facing the same price $P(x)$. (ii) Denote the total output of good l by $q = \sum_j q_j$. We assume that any aggregate output q is produced efficiently across all firms. That is, $c'_j(q_j) = C'(q)$ for all j . This is assured if all firms are price takers facing the same prices. Now

$$\begin{aligned} dS &= \sum_{i=1}^I \phi'_i(x_i) dx_i - \sum_{j=1}^J c'_j(q_j) dq_j \\ &= P(x) \sum_{i=1}^I dx_i - C'(q) \sum_{j=1}^J dq_j \\ &= P(x) dx - C'(q) dq. \end{aligned}$$

But since $x = q$, the total consumption equals the total production of good l , we have

$$dS = P(x)dx - C'(x)dx.$$

Therefore,

$$\begin{aligned} S &= S_0 + \int_0^x P(s)ds - \int_0^x C'(s)ds \\ &= S_0 + \int_0^x [P(s) - C'(s)]ds. \end{aligned}$$

Here $\int_0^x [P(s) - C'(s)]ds$ represents the area above the inverse supply curve and below the inverse demand curve from with the quantity varying from 0 to x . S_0 is a constant equal to the social surplus when $x = 0$.

Notice that the social surplus is maximized at x^* where $P(x^*) = C'(x^*)$, which is the competitive equilibrium output. This, of course, is a restatement of the first welfare theorem.

Two other useful concepts in welfare analysis are aggregate consumer surplus and aggregate producer surplus. If the effective price for good l faced by consumers is \hat{p} , which implies the aggregate consumption of good l is $x(\hat{p})$, the aggregate consumer surplus is defined as the gross consumer benefits from consuming good l minus the consumers' total expenditures on this good:

$$CS(\hat{p}) = \sum_i \phi_i(x_i(\hat{p})) - \hat{p}x(\hat{p}).$$

Since

$$d\left[\sum_i \phi_i(x_i(\hat{p}))\right] = P(x(\hat{p}))dx(\hat{p}),$$

and

$$\int_0^{\sum_i \phi_i(x_i(\hat{p}))} ds = \int_0^{x(\hat{p})} P(s)ds,$$

we obtain

$$\sum_i \phi_i(x_i(\hat{p})) = \int_0^{x(\hat{p})} P(s)ds.$$

Thus,

$$\begin{aligned} CS(\hat{p}) &= \int_0^{x(\hat{p})} P(s)ds - \hat{p}x(\hat{p}) \\ &= \int_0^{x(\hat{p})} [P(s) - \hat{p}]ds. \end{aligned}$$

Now, let $s = x(z)$, then

$$\begin{aligned} CS(\hat{p}) &= \int_{x^{-1}(0)}^{\hat{p}} (z - \hat{p})dx(z) \\ &= (z - \hat{p})x(z) \Big|_{x^{-1}(0)}^{\hat{p}} + \int_{\hat{p}}^{\infty} x(z)dz \\ &= \int_{\hat{p}}^{\infty} x(s)ds \end{aligned}$$

By similar derivation, the aggregate profit, or aggregate producer surplus, when firms face effective price \hat{p} , is

$$\begin{aligned} \Pi(\hat{p}) &= \Pi_0 + \int_0^{q(\hat{p})} [\hat{p} - C'(s)]ds \\ &= \Pi_0 + \int_0^{\hat{p}} q(s)ds. \end{aligned}$$

Example 3 *The welfare effects of a distortionary tax. Suppose that a sales tax $t > 0$ is levied on consumers for purchasing each unit of good l , but the tax revenue is returned to consumers through lump-sum transfers. What are the welfare effects of this tax policy?*

Let the equilibrium with tax be $(x_1^*(t), \dots, x_I^*(t), q_1^*(t), \dots, q_J^*(t))$ and $p^*(t)$. Let $x^*(t) = \sum_i x_i^*(t)$, and $q^*(t) = \sum_j q_j^*(t)$. Then the welfare change caused by the tax is equal to

$$S^*(t) - S^*(0) = \int_{x^*(0)}^{x^*(t)} [P(s) - C'(s)]ds,$$

which is negative since $x^*(t) < x^*(0)$ and $P(x) > C'(x)$ for $x < x^*(0)$. Thus social welfare is maximized by setting $t = 0$. When $t > 0$, the loss is called the deadweight loss from distortionary tax.

We can also find the change in aggregate consumer surplus and aggregate producer surplus caused by the tax policy.

$$\begin{aligned} CS(p^*(t) + t) - CS(p^*(0)) &= \int_{p^*(t)+t}^{\infty} x(s)ds - \int_{p^*(0)}^{\infty} x(s)ds \\ &= - \int_{p^*(0)}^{p^*(t)+t} x(s)ds. \end{aligned}$$

$$\Pi(p^*(t)) - \Pi(p^*(0)) = - \int_{p^*(t)}^{p^*(0)} x(s)ds.$$

The tax revenue to the government is $tx^*(t)$. The deadweight loss is

$$DW = - \int_{p^*(0)}^{p^*(t)+t} x(s)ds - \int_{p^*(t)}^{p^*(0)} x(s)ds + tx^*(t).$$

Free-Entry and Long-Run Competitive Equilibria

We now consider a competitive market with free entry (and exit). A competitive equilibrium with free entry is often called a long-run competitive equilibrium. Suppose that an infinite number of potential firms has access to a technology for producing good l with cost function $c(q)$ and $c(0) = 0$. The aggregate demand is $x(p)$, where p is the market price. Each firm's output is q , the number of firms in the industry is J , and total industry output is Q .

Definition 4 *Given an aggregate demand function $x(p)$ and a cost function $c(q)$ for each potentially active firm with $c(0) = 0$, a triple (p^*, q^*, J^*) is a long-run competitive equilibrium if*

- (i) q^* solves $\text{Max}_{q \geq 0} p^*q - c(q)$ (Profit maximization)
- (ii) $x(p^*) = J^*q^*$ (Demand = Supply)
- (iii) $p^*q^* - c(q^*) = 0$ (Free entry condition)

Let $q(p)$ be each firm's supply correspondence, $\pi(p)$ be each firm's profit, then the long-run aggregate supply correspondence is

$$Q(p) = \begin{cases} \infty & \text{if } \pi(p) > 0 \\ \{Q \geq 0 : Q = Jq \text{ for some integer } J \geq 0 \text{ and } q \in q(p)\} & \text{if } \pi(p) = 0 \end{cases}$$

If p is such that $\pi(p) > 0$, there will be infinitely many firms each producing a strictly positive amount of output. p^* is a long-run competitive equilibrium price if and only if $x(p^*) \in Q(p^*)$.

If $c(q) = cq$ for some $c > 0$ (constant returns to scale), then we must have $p^* = c$, $J^*q^* = x(c)$, with J^* and q^* being indeterminate. (Why? From (i), $p^* \leq c$. With the assumption $x(c) > 0$, we must then have $q^* > 0$. From (iii), $(p^* - c)q^* = 0$.) The supply correspondence of each firm is

$$q(p) = \begin{cases} \infty & \text{if } p > c \\ [0, \infty) & \text{if } p = c \\ 0 & \text{if } p < c \end{cases},$$

and the aggregate supply correspondence takes the same form.

If $c(q)$ is increasing and strictly convex (decreasing returns to scale), and assuming $x(c'(0)) > 0$, then no long competitive equilibrium exists. The reason is as follows: If $p > c'(0)$, then $\pi(p) > 0$. If $p \leq c'(0)$, then $Q = 0$ while $x(p) > 0$. In a graph, the aggregate demand curve has no intersection with the graph of the aggregate supply correspondence

$$Q(p) = \begin{cases} \infty & \text{if } p > c'(0) \\ 0 & \text{if } p \leq c'(0) \end{cases}$$

In order to obtain a long-run competitive equilibrium with a unique number of firms, the long-run cost function must be such that there exists some output under which the average cost is minimized. Let \bar{q} be the output that minimizes $\frac{c(q)}{q}$, and $\bar{c} = \frac{c(\bar{q})}{\bar{q}}$. Assume $x(\bar{c}) > 0$. Then there is a unique long-run competitive equilibrium

where $p^* = \bar{c}$, $q^* = \bar{q}$, and $J^* = \frac{x(\bar{c})}{\bar{q}}$. To see this, first notice that if $p > \bar{c}$, then $\pi(p) > 0$. Next, if $p < \bar{c}$, $\pi(p) = pq - q(c(q)/q) \leq pq - q\bar{c} = q(p - \bar{c}) < 0$. Thus at any long-run equilibrium we must have $p^* = \bar{c}$. Now if $p^* = \bar{c}$, profit maximization implies $q^* = \bar{q}$, and the free entry condition is also satisfied. Finally, the demand = supply condition is satisfied by setting $J^* = \frac{x(\bar{c})}{\bar{q}}$. The long-run aggregate supply correspondence is

$$Q(p) = \begin{cases} \infty & \text{if } p > \bar{c} \\ \{Q \geq 0 : Q = J\bar{q} \text{ for some } J \geq 0\} & \text{if } p = \bar{c} \\ 0 & \text{if } p < \bar{c} \end{cases}$$

Note that when $c(q)$ is differentiable, \bar{q} satisfies the f.o.c. of minimizing $\frac{c(q)}{q}$:

$$c'(\bar{q})\bar{q} - c(\bar{q}) = 0,$$

or

$$c'(\bar{q}) = \frac{c(\bar{q})}{\bar{q}}.$$

The potential problem here is that J^* may be small, or it may not be an integer. When $x(\bar{c})$ is sufficiently large relative to \bar{q} , however, we can ignore the integer problem.

The long-run and short-run cost functions are generally different. In the long run, firms can enter and exit the market freely, while in the short run a firm may not be able to exit the market freely. One way to model such a difference is to have the long-run cost function as

$$c(q) = \begin{cases} K + \psi(q) & \text{if } q > 0 \\ 0 & \text{if } q = 0 \end{cases}$$

and the short-run cost function as

$$c_s(q) = K + \psi(q) \text{ for all } q \geq 0.$$

Another difference between long- and short-runs is that some inputs may not be adjustable in the short run. In this case, a firm's inputs choice may not be optimal in the short run following a change in its output.

The differences in the long-run and short-run costs lead to different comparative statics in the long run and the short run. A positive demand shock, for instance, would raise prices and result in positive profits for firms in a competitive market in the short run, but in the long run, as new firms enter the market, the market price tends to fall back to the pre-shock level and firms again earn zero profit in equilibrium.