Vertical integration, exclusive dealing, and \textit{ex post} cartelization

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and

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Abstract. This paper uncovers an unnoticed connection between exclusive contracts and vertical organization. A vertically integrated firm can use exclusive dealing to foreclose an equally efficient upstream competitor and to cartelize the downstream industry. Neither vertical integration nor exclusive dealing alone achieves these anticompetitive effects. The cartelization effect of these two practices may be limited when downstream firms are heterogeneous and supply contracts are not contingent on uncertain market conditions. The extent of cartelization also depends on the degree of downstream market concentration and on the degree to which downstream competition is localized.

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1. Introduction

Antitrust scholars have devoted much ink to the competitive effects of vertical mergers (Riordan and Salop, 1995). For the most part, the economics literature focuses on how vertical integration \textit{per se} alters pricing incentives in relevant upstream and downstream markets. The Chicago school of antitrust, represented by Bork (1978), emphasizes that the efficiencies of vertical integration are likely to cause lower prices to final consumers, while a more recent strategic approach to the subject, represented by Ordover, Salop and Saloner (1990) and Hart and Tirole (1990), shows how vertical integration lacking any redeeming efficiencies might have the opposite purpose and effect. Ma (1997), Choi and Yi (2000), Church and Gandal (2000), and Chen (2001) consider richer models that feature trade-offs between anticompetitive effects and efficiencies. The debate is far from settled, in no small part because workable \textit{indicia} of harmful vertical mergers are lacking except in special cases (Riordan, 1998).

The use of exclusive contracts by customers and suppliers in intermediate product markets is equally controversial. The courts and antitrust agencies historically have treated exclusive dealing harshly, finding in many cases such practices illegally to foreclose competition. The Chicago school disputes this approach, advising instead that exclusive contracts are presumptively efficient, because usually it is unprofitable to foreclose competition \textit{via} exclusive contracts without good efficiency reasons (Bork, 1978). More recently, industrial organization economists have studied alternative models that demonstrate equilibrium incentives to foreclose more efficient potential entrants with exclusive contracts (Aghion and Bolton, 1987; Bernheim and Whinston, 1988; Rasmusen, Ramseyer and Wiley, 1991; Segal and Whinston, 2000; Spector, 2004).

An important institutional feature of some intermediate product markets is the coexistence of vertical integration and exclusive contracts. For instance, in \textit{Standard Oil Co. v. U.S.} (1949), Standard Oil sold about the same amount of gasoline through its own service stations as through independent retailers with which it had exclusive dealing contracts. In \textit{Brown Shoe Co. 62 F.T.C. 679} (1963), Brown Shoe had vertically integrated into the re-
etailing sector while using exclusive dealing contracts with independent retailers. In *U.S. v. Microsoft* (D.D.C. 2000), Microsoft’s had license agreements with competing online service providers, requiring them to promote and distribute Microsoft’s Internet Explorer to the exclusion of competitive browsers. This institutional feature is potentially important because, as we shall show, the incentive for and effects of exclusive contracts may depend on whether an upstream supplier is vertically integrated, and, conversely, the returns to vertical integration may depend on the possibility of exclusive contracting.

While the existing economics literatures on vertical integration and exclusive contracts yield important insights on the competitive effects of these practices used in isolation, the literatures generally ignore incentives for and effects of these practices in combination. The purpose of this paper is to uncover an unnoticed connection between exclusive contracts and vertical integration, and to develop a model for analyzing how these practices complement each other to achieve an anticompetitive effect. More specifically, we argue that a vertically integrated upstream firm has the ability and incentive to use exclusive contracts to exclude equally efficient upstream competitors and control downstream prices. The *ex post* effect is a cartelization of the downstream industry. Neither exclusive dealing nor vertical integration alone has this anticompetitive effect.

The paper is organized as follows. Section 2 previews our basic ideas. We illustrate the relationship between vertical integration and exclusive dealing in a simple model of industrial organization with two identical upstream and two identical downstream firms. We then discuss complications that arise with heterogeneous downstream firms when competitive advantage is uncertain and non-contractible, thus providing a transition to our main model of bilateral duopoly. The main model studied in Section 3 combines upstream requirements contracting with downstream bidding to serve final customers. In this model, one or the other downstream firm has an *ex post* competitive advantage in selling to a

\[ \text{As discussed later, the Hart and Tirole (1990) model explains the exclusion of only a less efficient competitor. While the Ordover, Salop, and Saloner (1990) model does demonstrate the equilibrium exclusion of an equally efficient competitor, some controversial assumptions of the model limit its applicability (Hart and Tirole, 1991; Reiffen, 1992; Ordover, Salop and Saloner, 1992).} \]
particular customer, but these downstream advantages are not contractible *ex ante* when requirements contracts are struck. We demonstrate in this context the ability of a vertically integrated firm profitably to employ an exclusive contract that raises input prices and cartelizes the downstream duopoly. We further show that exclusive contracts do not achieve this anticompetitive effect if the industries are vertically separated, and discuss extensions to bilateral oligopolies, and to private rather than public requirements contracts. Section 4 reviews some key features of our model of downstream competition, discusses the robustness to other models of downstream competition, relates our results to previous economics literature, discusses two relevant antitrust cases, and discusses policy implications.

An Appendix proves the formal results of Section 3. A separate Web Appendix, available at http://www.rje.org/main/Sup-mat.html, discusses two alternative spatial models of downstream markets with multiple independent competitors: a hub-and-spokes model of non-localized competition, and a circle model of localized competition. The results obtained earlier extend naturally to these models of bilateral oligopoly, with additional insights that the extent of upstream foreclosure and downstream cartelization depends on the nature of competition and on concentration in the downstream market.

2. Basic ideas

That vertical integration and exclusive dealing can combine to foreclose an equally efficient upstream competitor and to raise downstream prices is easy to demonstrate in a simple model of industrial organization. Suppose there are two identical upstream firms, \( U_1 \) and \( U_2 \), and two identical downstream firms, \( D_1 \) and \( D_2 \). The downstream firms require one unit of an intermediate good to produce one unit of the final good, for which identical consumers have a known reservation price \( V \). Upstream costs are normalized to zero and downstream costs per unit of production are equal to \( C < V \). If the firms are independent, then Bertrand competition in the upstream market followed by Bertrand competition in the downstream market results in a final goods price equal to \( C \). Against this backdrop, a vertically integrated \( U_1-D_1 \) has an incentive to purchase an exclusive right to serve the
downstream market and charge final consumers a price equal to $V$. For example, $U1-D1$ might pay $D2$ to withdraw from the market, or, alternatively, acquire $D2$. Such blatant monopolization likely would meet objections from antitrust authorities. More benign in appearance is an exclusive requirements contract that achieves the same anticompetitive effect. A contract that requires $D2$ to purchase from $U1$ at a price of $V - C$ fully extracts monopoly rents from the downstream market. Firm $U2$ is excluded from the upstream market, and final consumers pay $V$ to purchase from either $D1$ or $D2$.

It is interesting that $D2$ does not need much persuasion to agree to purchase its requirements exclusively from $U1-D1$ on non-competitive terms. If $D2$ were to decline an exclusive requirements contract with $U1-D1$, and instead to deal with $U2$ on competitive terms, then vigorous competition from $D1$ would squeeze out downstream profits to the point where $D2$ would be happy to have fallen into $U1$’s exclusive arms for a small concession, e.g. a small fixed fee. The Chicago school correctly observes that a downstream firm must be compensated to agree to forgo the benefits of upstream competition (Bork, 1978), but the above simple model shows that the necessary compensation need not be large if the firm has little to lose because of vigorous downstream competition. An exclusive contract effectively monopolizes the downstream industry, and the monopoly rents can be shared in some measure by all concerned firms.

It also is interesting that neither vertical integration nor exclusive dealing alone can be counted on to achieve these anticompetitive effects if contracts are bilateral. The vertically integrated $U1-D1$ could not persuade the independent $D2$ to pay a supra-competitive price for the intermediate good without an exclusive contract, because $D2$ would retain an ex post incentive to purchase from $U2$ on competitive terms and cut its retail price to steal business from $D1$. Similarly, unable to commit to a multilateral contract that binds both $D1$ and $D2$, a vertically-separated $U1$ is unable to pay $D1$ and $D2$ enough to induce them both

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2The presence of additional equally efficient stand-alone upstream firms does not change the argument. Furthermore, if there are additional equally efficient stand-alone downstream firms, $U1-D1$ can offer the same requirement contracts to all stand-alone downstream firms and achieve the monopoly outcome.

3In formalizing and qualifying Bork’s argument, Bernheim and Whinston (1998) ignore downstream competition and vertical integration in their models of exclusive dealing.
independently to forego the competitive alternative. Thus, vertically-separated upstream firms in equilibrium maximize bilateral profits by offering each downstream firm an efficient two-part tariff that sets the unit price of the intermediate good equal to marginal cost.\footnote{If upstream contracts are public commitments, then there exists a continuum of subgame perfect equilibria in which \( U1 \) and \( U2 \) both offer the intermediate good to \( D1 \) at a price equal to 0 (marginal cost) and to \( D2 \) at a price \( W \geq 0 \), and \( D1 \) sells the final good at a price \( \min(W + C,V) \). The competitive equilibrium with \( W = 0 \) seems the most natural one. This multiplicity of equilibria is an artifact of the homogeneous downstream market structure, and vanishes in our main model with heterogeneous downstream firms. The multiplicity also vanishes in the case of private bilateral contracts (Chen and Riordan, 2003).}

Matters are more complicated if downstream market conditions are uncertain and non-contractible. Suppose that \( C \) is a random variable, and that the realization of \( C \) becomes known after contracting for the intermediate good, but before setting downstream prices. Suppose further that requirements contracts take the form of uncontingent two-part tariffs. Then monopolization of the downstream industry by \( U1-D1 \) is accomplished with an exclusive requirements contract that excludes \( D2 \) by setting the marginal price of the intermediate good above all possible values of \( V - C \). Otherwise, competition from \( D2 \) would drive the downstream price below the monopoly level in some states of the world. Thus, under conditions of uncertainty and non-contractibility, \( U1-D1 \) can use an exclusive contract effectively to purchase a monopoly right. The contract is hardly subtle, and such blatant exclusion likely would catch the attention of antitrust authorities.

Matters are complicated further by downstream heterogeneity. If some consumers prefer \( D2 \)'s product, or are more cheaply served by \( D2 \), then a requirements contract that excludes \( D2 \) obviously cannot fully maximize industry joint profits. Rather a fully effective \emph{ex post} cartelization of the downstream industry would require coordinated pricing that divides the downstream market efficiently. For example, if random downstream costs have different realizations for \( D1 \) and \( D2 \), then it is efficient to assign final consumers to the low cost firm. But if these uncertain downstream market conditions are non-contractible, then \( U1-D1 \) would have the conflicting incentives both to exclude and not to exclude \( D2 \). \( U1-D1 \) generally is unable both to divide the market efficiently and to fully extract rents with a two-part tariff that \( D2 \) would accept. Thus, the combination of uncertainty, non-
contractibility, and heterogeneity appear to create difficulties for *ex post* cartelization via vertical integration and exclusive dealing.

To understand fully the relationship between vertical integration and exclusive dealing, therefore, it is important to go beyond the simple case of homogeneous downstream firms and to study the relationship under conditions of downstream heterogeneity, uncertainty, and noncontractibility. In what follows, we analyze a game-theoretic model of an industry possessing these features. This analysis will make clear several points. First, the synergistic relationship between vertical integration and exclusive dealing is not due to the extremely vigorous nature of potential downstream competition between identical producers; rather, it holds more generally in the presence of heterogeneous downstream firms who possess some degree of market power. Second, while the vertically integrated firm has the incentive and ability to exclude upstream competition and cartelize the downstream market, its ability to do so may be reduced with downstream heterogeneity and noncontractible uncertainty. In particular, the fixed payment needed to persuade $D2$ to enter the exclusive contract may not be small when downstream firms are heterogeneous, and only partial cartelization of the downstream industry is feasible when downstream monopoly prices vary with non-contractible market conditions. Third, extending the model to multiple independent downstream competitors, while maintaining the assumption of bilateral contracting, reveals that the degree of *ex post* cartelization of the downstream industry depends on market concentration and on whether or not competition is localized. Fourth, the exclusive contracts that a vertically integrated firm uses to cartelize the downstream industry are not blatant antitrust violations. The vertically integrated firm subtly employs the marginal wholesale price of a two part tariff to raise the downstream price, and judicially employs the fixed fee to distribute the rents from cartelization.

\[\text{This is despite a hidden bonus to } D2: \text{ Because the integrated firm treats foregone wholesale revenues as an opportunity cost, both of the downstream firms offer the final good at supra-competitive prices, which provides another source of compensation to } D2 \text{ for agreeing to the exclusivity.}\]
3. Heterogeneous downstream firms

In this section, we study the main model of the paper. After introducing the model, we consider a benchmark case in which an upstream monopolist is vertically integrated with one of the downstream duopolists. We then introduce an equally efficient non-integrated upstream competitor, and prove that in equilibrium the vertically integrated firm profitably employs an exclusive contract to achieve the same market outcome as in the upstream monopoly case, except for the distribution of rents between the upstream and downstream industries. We further show that exclusive contracts are irrelevant if the industries are vertically separated. We complete this section by discussing what happens if the model is extended to allow multiple independent downstream firms and if contracts are private rather than public.

Main model

We model a vertically organized industry with $M$ customers purchasing a product from two downstream firms $D1$ and $D2$. Shortly, we will simplify without loss of generality by setting $M = 1$. The downstream firms manufacture differentiated products by combining a component input with other inputs whose cost is normalized to zero. There are two upstream firms $U1$ and $U2$ supplying the component input at the same unit cost $c_0$. For example, the customers could be construction companies purchasing ready-mixed concrete from a downstream industry that procures cement from an upstream industry. Alternatively, the customers could be automobile manufacturers purchasing batteries from a downstream industry that procures battery separators (a component input) from an upstream industry. Still another example could be firms purchasing health services on behalf of employees from health maintenance organizations who use upstream hospital facilities. We will refer to a customer alternatively as a "consumer", even though the model perhaps best applies to business-to-business transactions.

The model is patterned roughly on markets for cement and concrete markets. Cement is a fixed proportions input into the production of concrete, and concrete producers typically procure cement supplies under requirements contracts. The demand for ready-mixed
concrete is located at construction sites that are difficult to predict or specify in contracts. Since delivered ready-mixed concrete requires a cement truck, transportation costs evidently are important and idiosyncratic to the location of the construction sites. The model captures these cost characteristics with a number of simplifying assumptions. We revisit cement and concrete markets at the end, when we discuss applications.

A consumer located at \( x \in [0, 1] \) is interested in purchasing one unit of the downstream product. The consumer’s uncertain reservation value \( v \) has a cumulative distribution function \( F(v) \) on support \( [\bar{v}, \tilde{v}] \), where \( 0 \leq \bar{v} < \tilde{v} < \infty \). The corresponding probability density function is \( f(v) > 0 \) for \( v \in [\bar{v}, \tilde{v}] \). The consumer’s uncertain reservation value gives rise to a downward-sloping expected demand curve.\(^6\) The corresponding expected marginal revenue function is also smooth and downward sloping under the following maintained familiar technical assumption:

\[
A1. \quad \frac{d \left( \frac{1 - F(p)}{f(p)} \right)}{dp} \leq 0.
\]

To sell to the consumer \( D1 \) incurs transportation costs \( \tau x \) and \( D2 \) incurs \( \tau (1 - x) \), where \( \tau > 0 \) is a fixed parameter, measuring the degree of ex post cost heterogeneity. Thus, the transportation costs of the two firms are negatively correlated. For simplicity the consumer’s location \( x \) is uniformly distributed on \([0, 1]\). This simple spatial cost structure captures adequately the more general idea of uncertain cost heterogeneity.

The downstream firms “bid” prices to the consumer, \( P_1 \) and \( P_2 \). The consumer purchases the lower priced product as long as that price is below the consumer’s realized reservation value \( v \), and nothing otherwise. At the time of bidding, the firms know \( x \) but do not know \( v \). Thus, each firm understands the extent to which it is advantaged or disadvantaged in competing for each consumer, while remaining uncertain about the magnitude of the consumer’s demand.\(^7\) As explained below, in equilibrium the advantaged downstream firm

\(^6\)We could replace the assumption of a random \( v \) with the assumption that the consumer has a conventional downward sloping demand curve.

\(^7\)The model can be reinterpreted as a discrete choice model of consumer demand in which the “transportation cost” is incurred directly by the consumer. Suppose each consumer has a discrete demand for the
predictably wins the competition. In the symmetric case \( x = \frac{1}{2} \), we select the equilibrium in which \( D1 \) wins.

Now suppose that \( U1 \) and \( D1 \) are vertically integrated. \( U1 \) and \( U2 \) each offer \( D2 \) a contract requiring \( D2 \) to purchase exclusively from \( U1 \) or \( U2 \). The requirements contracts are assumed to take the form of a two-part tariff specifying a fixed transfer payment from \( D2 \) to \( Ui, t_i \), and a unit price \( r_i \) that \( D2 \) pays contingent on actual production. Thus \( t_i > 0 \) means that \( D2 \) pays a fee to \( Ui \), while \( t_i < 0 \) means the opposite. If a contract is accepted, \( t_i \) is paid irrespective of whether any sale is made, but \( r_i \) is paid only if \( D2 \) actually makes a sale. Thus the component is produced to order, i.e. the exclusive supplier produces the component only if \( D2 \) succeeds in the downstream market.

At the time of contracting between the upstream and downstream industry, customer characteristics are uncertain. A consumer’s location \( x \) becomes known after \( D2 \) commits to an exclusive supply relationship, but before downstream price competition. Moreover, customer characteristics, \( x \) and \( v \), are not contractible. The implicit assumption justifying this approach is that the transaction costs for determining the realization of \( x \), and making the contract depend on this determination are prohibitively high. The consumer’s reservation value is never observed publicly, although it is easy to write a contract contingent on production resulting from the consumer’s purchase decision. Furthermore, \( U1-D1 \) cannot commit to any internal transfer price that is not \textit{ex post} jointly optimal, nor can anyone commit to a retail price through the supply contracts.

So far, we have assumed an arbitrary finite number of customers \( M \). It is sufficient, downstream product determined by a pair of values \((v_1, v_2) = (v - \tau x, v - \tau (1 - x))\). Then, the variable \( x \) determines product differentiation from the perspective of the customer, while the variable \( v \) determines the customer’s average value for the downstream product. It is standard in discrete choice oligopoly models to decompose consumer valuations into a component that is known to firms and a component that is unknown. The distinguishing feature of our model is that the unknown component is a common value.

\(^8\) A conceivable possibility is that contract terms depend on messages exchanged after \( x \) is realized, in the spirit of the Nash implementation literature (Maskin, 1985). We implicitly assume that the transactions costs associated with the necessary message game are prohibitively burdensome. Alternatively, such communication between downstream competitors might be construed to violate the antitrust laws. In any case, the usual message game \textit{a la} Maskin would violate the assumption that contracts are bilateral.
however, to focus on the case $M = 1$. This is because the downstream firms compete separately for the business of each customer, and the only thing that links these competitions is the contract price per unit that determines the marginal cost of $D2$. Because the customers are identical \textit{ex ante}, the equilibrium $r_i$ are independent of $M$, and the equilibrium $t_i$ are proportional to $M$. Therefore, we simplify the exposition henceforth by setting $M = 1$. The main results extend to an arbitrary $M$ by interpreting the $t_i$ as payments per potential customer.\footnote{We could also assume $M$ is random, and interpret $t_i$ as the transfer payment divided by the expected number of customers in the market.}

To summarize, the timing of the game (assuming $M = 1$) when $U1$ and $D1$ are vertically integrated is as follows:

Stage 1. $U1$ and $U2$ offer contracts $(t_1, r_1)$ and $(t_2, r_2)$.

Stage 2. $D2$ chooses a contract.

Stage 3. $x$ is realized.

Stage 4. $D1$ and $D2$ bid prices.

Stage 5. $v$ is realized and the consumer makes a purchase decision.

Remark 1. The game form ignores the possibility that $D2$ might decline any exclusive contract and instead purchase on a spot market after learning $x$. A spot market is irrelevant because, as will be seen shortly, in equilibrium $U2$ offers a requirements contract on terms that are the same as would prevail in the spot market.

We refine equilibrium by requiring that $D1$ and $D2$ do not set prices below their costs at Stage 4, and $U2$ does not offer a contract at Stage 1 that would be unprofitable if accepted by $D2$. Thus we confine our attention to equilibrium strategies with the property that a player never strictly prefers her offer to be rejected, whether in Stage 1 or in Stage 4 of the game. The refinement prevents firms from "squeezing" the profits of rivals by making an offer that are not credible. The refinement is implied by the stronger requirement that players do not use weakly dominated strategies, which is too strong for our purposes because it would eliminate all pure strategy equilibria.\footnote{This is a familiar problem in games with continuous strategy spaces, e.g. the Bertrand duopoly with...
Several features of the model are worth previewing and highlighting. First, cost heterogeneity and uncertainty are captured by the random location variable $x$. The cost heterogeneity is manifest in an asymmetric Bertrand model of downstream competition in which one firm or another has a clear advantage. When a firm enjoys a small advantage, the rival imposes a competitive constraint by setting price equal to its opportunity cost of supplying the consumer. The advantaged firm must meet that price in order to make the sale. A firm with a sufficiently large advantage, however, is effectively a monopolist. Second, the asymmetric Bertrand model of downstream competition brings into sharp focus the idea that foregone upstream revenues are an opportunity cost of $U1-D1$ when $U1$ has an exclusive supply contract with $D2$. In this case, an increase in $r1$ softens downstream competition whenever the disadvantaged rival imposes a competitive constraint by increasing dollar for dollar the marginal opportunity cost of both $D2$ and $U1-D1$. By eliminating this opportunity cost for $U1-D1$, a decision by $D2$ to contract with $U2$ instead of $U1$ causes a "price war" in the downstream market. Third, the non-contractibility of downstream costs is captured by the assumption that requirements contracts are not contingent on the realization of $x$. This non-contractibility creates a conflict between horizontal control and vertical control for the vertically integrated firm. Horizontal control means that $U1-D1$ would like to raise $r1$ in order to soften competition when the rival poses a competitive constraint. Vertical control means that $U1-D1$ would like to lower $r1$ in order to avoid double marginalization when the independent firm has monopoly power. The conflict means that $U1-D1$ cannot perfectly cartelize the downstream industry with an exclusive requirements contract. Fourth, the random $v$ is merely a device to construct a downward sloping demand curve, which is important for "cartelization" to have a harmful effect on social welfare. Fifth, for convenience we assume that contracts are public. Our main results hold under private bilateral contracting as well, as we demonstrated in our earlier cost asymmetry (Kreps, 1990, pp. 419, footnote d). An alternative approach is to consider a sequence of price games with discretized strategy spaces (prices), the limit of which is our game as the mesh of the partition goes to zero. An undominated equilibrium exists for each game in the sequence, as well as for the limit. This would then preserve equilibrium existence in our game under the dominance refinement.
working paper (Chen and Riordan, 2003).\(^\text{11}\)

**Upstream monopoly**

We begin by considering a benchmark situation where \(U_1\) is the only supplier in the upstream market, and modify the game accordingly. Thus \(U_1\) offers \((t, r)\) to \(D_2\) at Stage 1, and \(D_2\) accepts or rejects at Stage 2. This benchmark model establishes some preliminary results for our analysis of upstream duopoly.

It is useful to characterize unconstrained monopoly prices in the downstream market. If \(D_2\) rejects \(U_1\)'s contract offer at Stage 1, or if the realization of \(x\) at Stage 3 is sufficiently favorable, then \(D_1\) operates as an unconstrained monopolist and chooses \(p = P_{m_1}(x)\) to maximize \(\{(p - c - \tau x)[1 - F(p)]\}\). Given the regularity assumption \(A1\), \(P_{m_1}(x)\) exists uniquely and satisfies:

\[
P_{m_1}(x) - c - \tau x = \frac{1 - F(P_{m_1}(x))}{f(P_{m_1}(x))}.
\]

Similarly, if \(D_2\) accepts \(U_1\)'s contract offer and is sufficiently advantaged by the realization of \(x\), then \(D_2\) offers the monopoly price \(P_{m_2}(x, r)\) satisfying

\[
P_{m_2}(x, r) - r - \tau (1 - x) = \frac{1 - F(P_{m_2}(x, r))}{f(P_{m_2}(x, r))},
\]

assuming \(r\) is not prohibitively high, i.e. \(r < \bar{v} - \tau (1 - x)\). It can be easily verified that \(P_{m_1}(x)\) increases in \(x\) and \(P_{m_1}(x) - c - \tau x\) decreases in \(x\). Similarly, \(P_{m_2}(x, r)\) increases in \(r\) and \((1 - x)\); and \(P_{m_2}(x, r) - r - \tau (1 - x)\) decreases in \(r\) and \((1 - x)\). For convenience, we make the following additional technical assumption:

\[
A2. \quad P_{m_1}^{m}(0) \geq c + \tau.
\]

\(A2\) is satisfied if the likely values of \(V\) are not too small relative to \(c + \tau\). The assumption implies that, if \(r = c\), then \(U_1\)-\(D_1\) and \(D_2\) always pose a competitive constraint on each other. This innocuous assumption simplifies our arguments in certain places, but is not crucial for our main results.

\(^{11}\)If binding multilateral contracts were feasible, then vertical integration would not be a necessary ingredient of cartelization (Mathewson and Winter, 1984).
Now suppose that $D2$ accepts $U1$’s contract offer $(t, r)$, and competes against $D1$ for customer $x$. The price subgame at Stage 4 is an asymmetric Bertrand game. When $x < \frac{1}{2}$, $D1$ has the advantage, and Bertrand competition means that $D1$ wins the competition by setting its price either at its monopoly level or at the marginal cost of $D2$, whichever is less. When $x > \frac{1}{2}$, $D2$ is the more efficient supplier. $D1$ stands ready to sell at its marginal opportunity cost $r + \tau x$, or, if $r + \tau x > P_{1m}^m (x)$, at its monopoly price $P_{1m}^m (x)$ so that the probability of a sale is not unprofitably low. In response, $D2$ sets its price either at $P_{2m}^m (x, r)$ or at $\min\{P_{1m}^m (x), r + \tau x\}$, whichever is smaller. Accordingly, equilibrium pricing strategies are as follows. Define:

$$P_1(x, r) = \min \{P_{1m}^m (x), r + \tau (1 - x)\}$$

$$(3)$$

$$P_2(x, r) = \min \{P_{2m}^m (x, r), \min \{P_{1m}^m (x), r + \tau x\}\}.$$  

$$(4)$$

Lemma 1. Assume that $U1$ is the sole upstream supplier. (1) Suppose that $P_{1m}^m \left(\frac{1}{2}\right) \geq r + \frac{1}{2} \tau$. Then the following is a Nash equilibrium of the $D1$-$D2$ pricing subgame: If $x \leq \frac{1}{2}$, then $D1$ offers $P_1(x, r)$, $D2$ offers $r + \tau (1 - x)$, and the customer selects $D1$. If $x > \frac{1}{2}$, then $D2$ offers $P_2(x, r)$, $D1$ offers $\min\{P_{1m}^m (x), r + \tau x\}$, and the customer selects $D2$. (2) If $(t, r)$ is an equilibrium contract between $U1$ and $D1$, then $P_{1m}^m \left(\frac{1}{2}\right) \geq r + \frac{1}{2} \tau$.

Proof. See the Appendix.

The equilibrium prices when competitive constraints bind are those for a standard Bertrand duopoly with different constant marginal costs, say $c_1 < c_2$, where the equilibrium price is $c_2$. Although both sellers charging a price $p \in (c_1, c_2)$ can also be supported as a Nash equilibrium, seller 2 would prefer not to be selected as the supplier at such a price. Thus, requiring that a seller should not strictly prefer to be rejected at the price it bids, the only equilibrium in our pricing game between $D1$ and $D2$ is the one characterized in Lemma 1. In what follows, we consider this as the unique (refined) equilibrium in the pricing subgame.

From Lemma 1, in equilibrium $P_{1m}^m \left(\frac{1}{2}\right) \geq r + \frac{1}{2} \tau$. This holds because otherwise $r$ would be so high that $D2$ is unable to serve some of the consumers who are located closer to $D2$ than to $D1$, which means that industry profit is not maximized. We note that this also
holds if $D2$ has some outside option for obtaining the input. This extension is relevant for the case of upstream competition considered later.

We next define:

$$
\Pi(r) = \int_0^{\frac{1}{2}} [P_1(x, r) - \tau x - c] \left[ 1 - F (P_1(x, r)) \right] dx \\
+ \int_{\frac{1}{2}}^1 [P_2(x, r) - \tau (1 - x) - c] \left[ 1 - F (P_2(x, r)) \right] dx 
$$

(5)

$$
t(r) = \int_{\frac{1}{2}}^1 [P_2(x, r) - \tau (1 - x) - r] \left[ 1 - F (P_2(x, r)) \right] dx 
$$

(6)

$\Pi(r)$ is the joint upstream-downstream industry profit when $D2$ contracts to purchase from $U1$ at unit price $r$, and $t(r)$ is the transfer price that fully extracts rents from the downstream industry. We now fully characterize the equilibrium of the game.

**Proposition 1.** The game where $U1$ is the only upstream supplier has a unique equilibrium. At this equilibrium, $U1$ offers $D2$ contract $(\hat{r}, \hat{r})$, which is accepted by $D2$, where

$$
\hat{r} = \arg \max_{c \leq r \leq 0} \{ \Pi(r) \}, \quad \hat{t} = t(\hat{r}) .
$$

$D1$ is the seller with price $P_1(x, \hat{r})$ if $x \leq \frac{1}{2}$, and $D2$ is the seller with price $P_2(x, \hat{r})$ if $x > \frac{1}{2}$. Furthermore, $c \leq P_1^m(0) - \tau < \hat{r} < P_1^m \left( \frac{1}{2} \right) - \frac{1}{2} \tau$.

**Proof.** See the Appendix.

The equilibrium contract has a cartelizing effect. By charging $D2$ a wholesale markup $(\hat{r} - c)$, $U1$ raises $D2$’s marginal cost directly, creating an incentive for $D2$ to raise its prices. Thus, $D2$ sells at a higher price when $x \geq 1/2$, and poses less of a competitive constraint on $D1$ when $x < 1/2$. The markup also raises $U1-D1$’s opportunity cost, creating an incentive for $D1$ to raise its prices and become less of a competitive constraint on $D2$ when $x \geq 1/2$ and $P_2^m(x, \hat{r}) > \hat{r} + \tau x$. The overall effect is to lessen horizontal competition in the downstream market and to reduce consumer welfare, relative to the situation where the wholesale price for $D2$ is $c$.

\[\text{\textsuperscript{12}}\text{It is important for our result that } U1-D1 \text{ takes an integrated view of its operations and coordinates}\]
The cartelization of the industry, however, is only partial, due to the assumption that $x$ is not contractible and the resulting conflict between horizontal and vertical control. Full cartelization requires a monopoly price for all values of $x$. To see this, first consider the consumer at $x = 1$, where

$$P_2(1, \hat{r}) = \min \{ P_2^m(1, \hat{r}), \min \{ P_1^m(1), \hat{r} + \tau \} \} > P_1^m(0)$$

since $P_2^m(1, \hat{r}) > P_2^m(1, c) = P_1^m(0)$, $P_1^m(1) > P_1^m(0)$, and $\hat{r} + \tau > P_1^m(0)$. Therefore, for consumers sufficiently close to $x = 1$, $P_2(x, \hat{r}) > P_1^m(1 - x)$, or the price is above the vertically-integrated industry monopoly level. Thus, there is a problem of double marginalization (Spengler, 1950) when cost heterogeneity is greatest. Next, consider consumers at or slightly below $x = \frac{1}{2}$. For these consumers, since $\hat{r} < P_1^m(1) - \frac{1}{2} \tau$ from Proposition 1, we have $P_1(x, \hat{r}) < P_1^m(x)$, or the price is below the vertically-integrated industry monopoly level, i.e. there is a problem of excessive horizontal competition when the downstream firms have similar costs.

The obstacle to full cartelization is non-contractibility, i.e. contract terms do not vary with the location of the final consumer. This fact creates a tension between improving vertical efficiency in some circumstances and intensifying horizontal competition in others. The conflict arises in our model from the downward-sloping expected demand curve generated by the consumer’s uncertain reservation price. A lower value of $\hat{r}$ causes lower downstream prices by reducing $D2$‘s marginal cost as well as $U1$-$D1$‘s marginal opportunity cost. Thus, $U1$ faces a trade-off in setting the price to $D2$. Reducing $\hat{r}$ alleviates $D2$‘s double marginalization problem at some locations, but also intensifies horizontal price competition elsewhere. If $\hat{r}$ is reduced, neither $U1$-$D1$ nor $D2$ can commit not to undercut each other for the consumer that is located closer to the rival. The problem is that downstream monopoly prices vary with the location of the consumer; and the single instrument $\hat{r}$ cannot achieve these prices in all circumstances.

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Upstream duopoly

We now return to the model where the upstream market is a duopoly. Recall that the contracts offered by $U1$ and $U2$ are denoted by $(t_1, r_1)$ and $(t_2, r_2)$. We argue in several steps that in equilibrium $U1$ wins the contract competition on terms that effectively cartelize the downstream industry. The first step shows that $U2$ always seeks to contract with $D2$ with $r_2 = c$. The second step establishes that $U1-D1$ always can profitably outbid $U2$ with $r_1 = \hat{r}$. The last step fully characterizes equilibrium contracts, confirming that $U1-D1$ is willing and able to compensate $D2$ for declining to contract with $U2$ on competitive terms.

The following lemma establishes that $U2$ and $D2$ have a joint incentive to negotiate $r_2 = c$. Raising $r_2$ above $c$ would cause $D2$ to price less aggressively, but would have no salutary strategic effect of softening $D1$’s competition. The reason why the strategic effect is absent in our model is that $D1$ prices either at cost or at its own monopoly price whenever $D1$ is a competitive constraint, and these clearly do not depend on $r_2$. Therefore, the only consequence for $U2$ and $D2$ of raising $r_2$ above $c$ is the adverse one of causing $D2$ to succeed less.

Lemma 2. When $U2$ is the contracted supplier of $D2$, $r_2 = c$ maximizes the joint profits of $U2$ and $D2$.

Proof. See the Appendix.

Second, maximized industry profits are higher when $D2$ contracts with $U1-D1$, implying that $U1-D1$ can always profitably outbid $U2$ for $D2$’s business. The maximized joint profits of $U1-D1$ and $D2$ is $\Pi(\hat{r})$. If $D2$ were to contract with $U2$ with $r_2 = c$, then industry profit would be $\Pi(c)$, half of which would go to $U1-D1$, and half shared by $U2$ and $D2$. Notice that, from Proposition 1, $\Pi(\hat{r}) > \Pi(c)$. The reason for this drop in industry profits from a mere switch of equally efficient suppliers is that $U1-D1$ no longer internalizes the opportunity cost of upstream’s sales to $D2$, and therefore has a discrete incentive to price more aggressively in the downstream market. Furthermore, since

$$\Pi(\hat{r}) - \frac{1}{2} \Pi(c) > \frac{1}{2} \Pi(c),$$
U1-D1 can compensate D2 for agreeing to purchase at $r_1 = \hat{r}$ instead of $r_2 = c$ (even with $t_2 = 0$), and still extract an additional monopoly profit. The credible threat of a price war helps discipline D2 from contracting with U2.

These observations lead directly to the conclusion that in equilibrium U1-D1 cartelizes the downstream industry with an exclusive requirements contract. It is optimal for U1-D1 to set $r_1 = \hat{r}$ and for U2 to set $r_2 = c$. Given these variable price terms, U1-D1 and U2 compete by bidding $t_1$ and $t_2$ respectively. This bidding competition has the essential structure of asymmetric Bertrand game. Consequently, in equilibrium D2 contracts with U1, with U2 offering $(0, c)$ and U1 offering $(t_1^*, \hat{r})$, and

$$t_1^* = \int_{\frac{1}{2}}^{1} [P_2(x, \hat{r}) - \tau(1 - x) - \hat{r}] [1 - F(P_2(x, \hat{r}))] dx - \int_{\frac{1}{2}}^{1} \tau(2x - 1) [1 - F(c + \tau x)] dx.$$ 

(7)

The transfer $t_1^*$ extracts the downstream profits from D2 except for the amount D2 could earn by declining the contract offer and procuring from U2 on competitive terms. By assumption A2, this amount is equal to

$$t(c) = \int_{\frac{1}{2}}^{1} \tau(2x - 1) [1 - F(c + \tau x)] dx$$

(8)

Finally, it is straightforward that $t_1^* < 0$, in compensation for $\hat{r} > c$.

**Proposition 2.** The game where the upstream market is a duopoly has a unique equilibrium. In equilibrium U2 offers D2 $(0, c)$ and U1 offers D2 $(t_1^*, \hat{r})$ with $t_1^* < 0$, D2 contracts with U1, and the downstream equilibrium outcome is the same as under upstream monopoly.

**Proof.** See the Appendix.

Thus, a vertically integrated firm is able to outbid a equally efficient stand-alone supplier for an exclusive relationship with a downstream competitor. When the integrated firm supplies D2 at a price above marginal cost, the former has less incentive to undercut D2 because of the opportunity cost of foregone input sales to D2.\(^{13}\) This dampening of horizontal competition explains U1’s advantage and ability to preempt U2 (Gilbert and Newbery, \(^{13}\)This “opportunity cost” idea appeared in Chen (2001), who finds that a vertically integrated firm can only exclude a less efficient supplier. A key reason for our more dramatic result is that we allow two-part
Because of downstream heterogeneity, the profitable exclusion of \(U2\) may nevertheless cost \(U1-D1\) a substantial amount. However, this cost approaches zero as the difference between \(D1\) and \(D2\) disappears, i.e. \(t^*_1 < 0\) and \(\lim_{\tau \to 0} \int_{\frac{1}{2}}^{1} \tau (2x - 1) [1 - F(c + \tau x)] \, dx = 0\) imply \(\lim_{\tau \to 0} t^*_1 = 0\).

The exclusion of upstream competition leads to higher downstream prices compared to when \(U2\) supplies \(D2\). The exclusivity of the contract clearly is important for the cartelization outcome under vertical integration. Since \(\hat{r} > c\), \(D2\) would want to purchase from \(U2\) \textit{ex post} as long as \(r_2 < \hat{r}\), and \(U2\) would be willing to cut \(r_2\) to as low as \(c\) to gain \(D2\)'s business. This implies that, if upstream firms cannot sign exclusive contracts with downstream firms, perhaps due to legal restrictions or to difficulties in contract enforcement, then the input price to \(D2\) must be set at \(r_1 = r_2 = c\), with \(t_1 = t_2 = 0\). Therefore:

\textit{Remark 2}. In the game where the upstream market is a duopoly, the cartelization of the downstream market can be achieved only if exclusive requirements contracts are feasible.

\textbf{Vertical separation}

Earlier, we showed that exclusive contracts used by a vertically integrated firm can achieve the market outcome of an upstream monopolist. To see that vertical integration is important for the cartelization effect of the exclusive contracts, we next consider a variation of our model in which \(U1\) and \(D1\) are vertically separated independent firms. We shall show that exclusive contracts are irrelevant in this case: the equilibrium input price for both downstream firms is \(c\).

The timing of the modified game is as follows:

\textbf{Stage 1.} \(U1\) and \(U2\) each offer separate contracts to \(D1\) and \(D2\).\textsuperscript{15}

\textbf{Stage 2.} \(D1\) and \(D2\) choose contracts.

\textsuperscript{14}While we have assumed for simplicity that \(U1\) and \(U2\) are equally efficient, the same logic would hold, and so would Proposition 2, if \(U2\) had a small efficiency advantage. In this case, however, \(U1-D1\) would have an incentive to “outsource” supplies of the input from the more efficient \(U2\).

\textsuperscript{15}To be consistent with our earlier analysis, we again assume that these are exclusive contracts requiring tariff contracts in the input market while Chen considers only uniform price contracts. In addition, Chen’s model is based on non-exclusive contracts, although he allows some possibility of lock-in due to switching costs.
Stage 3. $x$ is realized.

Stage 4. $D1$ and $D2$ bid prices.

Stage 5. $v$ is realized and the consumer makes a purchase decision.

A contract offer from $Ui$ to $Dj$ is a transfer payment and intermediate goods price, $(t_{ij}, r_{ij})$ for $i, j = 1, 2$. Adapting our notation, we let $(t_j, r_j)$ now denote any contract that $Dj$ accepts, whether offered by $U1$ or $U2$. When the input prices for $D1$ and $D2$ are $(r_1, r_2)$, let the joint profits of $Dj$ with its supplier $Ui$ be $\Pi_{ij} (r_1, r_2)$, $i, j = 1, 2$. We further let

$$P(x, r_1, r_2) = \min \{P^m(x, r_1), r_2 + \tau (1 - x)\}$$

where $P^m(x, r)$ is the monopoly price for $D1$ to serve a consumer at marginal cost $(r + \tau x)$, defined implicitly by

$$P^m(x, r) - r - \tau x = \frac{1 - F(P^m(x, r))}{f(P^m(x, r))}.$$ 

If $[r_2 + \tau (1 - x)]$ is the marginal cost of $D2$, then equilibrium prices are $\max \{P(x, r_1, r_2), r_1 + \tau x\}$ for $D1$, and $\max \{P(1 - x, r_2, r_1), r_2 + \tau (1 - x)\}$ for $D2$. Bertrand competition implies that the downstream firm with the lowest marginal cost wins the customer. Thus, if $(r_1 + \tau x) \leq [r_2 + \tau (1 - x)]$, the equilibrium outcome is for $D1$ to serve consumer $x$ at price $P(x, r_1, r_2)$.

**Proposition 3.** The game under vertical separation has a unique equilibrium outcome where $(t^*_j, r^*_j) = (0, c)$ for both $j = 1, 2$.

**Proof.** See the Appendix.

If both $D1$ and $D2$ were to contract only with $U1$ at input prices above $c$, then downstream prices would be higher and joint upstream-downstream industry profits would also be higher. Therefore, one might conjecture that in equilibrium $U1$ would be able to use exclusive contracts to cartelize the downstream industry as in the case of vertical integration. So why is this not the case in the absence of vertical integration? The reason is that one of the downstream firms can pair with $U2$ at a lower input price and obtain a joint profit that a downstream firm to purchase only from a certain upstream firm, although exclusive contracts are not necessary for our result that the intermediate-good price will be equal to $c$ under vertical separation.
is more than its joint profit with $U1$ under the higher input price. This competitive option would frustrate any attempt by $U1$ to use exclusive contracts to cartelize the downstream industry, because it makes it too costly for an independent $U1$ to gain the compliance of both downstream firms.\footnote{In a repeated game setting, a nonintegrated $U1$ might use the threat of reversion to competitive pricing to cartelize the downstream market with exclusive contracts. If $D2$ were to reject $U1$’s exclusive deal, and contract with $U2$ instead on competitive terms, then $U1$ could punish the defection by contracting with $D1$ on competitive terms in the next period. If the period length were sufficiently short, then the subgame perfect equilibrium threat could support cartelization even in a vertically separated setting. Even in a repeated game setting, however, vertical integration can facilitate collusion (Nocke and White, 2003).} This reasoning is made precise in the Appendix.

Notice that under vertical integration, only $D2$ needs to be prevented from deviation, and $D1$’s pricing strategies depend on whether $D2$ purchases from $U1$ at $r_2 > c$. If $D2$ contracts to purchase from $U1$ at $r_2 > c$, $D1$ would price less aggressively in the downstream market, which leads to a higher joint upstream-downstream profits. If instead $D2$ contracts to purchase from $U2$ at input price $c$, then both $D1$ and $D2$ will compete with marginal cost $c$, resulting in lower upstream-downstream joint profits. Thus $U2$ is unable to attract $D2$ with a profitable deviation.

Since $r_i = c$ for $i = 1, 2$, there is no need for exclusive contracts in equilibrium, and firms have equilibrium incentives to negotiate supply arrangements on competitive terms, i.e. exactly as they would in spot markets.

Remark 3. When $U1$ and $D1$ are vertically separated, exclusive contracts are irrelevant in equilibrium, with a competitive upstream outcome.

**Extending to multiple downstream firms**

Our spatial model of downstream price competition is restrictive in that it only suits the case of bilateral duopoly. The logic of our results, however, is more general. In the Web Appendix, we first introduce a generalization of the model, in which $n$ downstream competitors are located at terminal nodes of a symmetric hub-and-spokes network and customers are randomly located on the connected spokes. The downstream firms incur transportation costs to deliver the intermediate good to a customer at a particular location.
Thus a firm located at the terminal node of a customer’s spoke has a cost advantage over other competitors. This hub-and-spokes model is interesting because it exhibits a strong form of non-localized competition;\textsuperscript{17} each downstream firm possesses market power over nearby customers, but is constrained by all other competitors who are equidistant.\textsuperscript{18} As in Chamberlinian monopolistic competition, each downstream firm perceives itself as competing against the rest of the market. The Web Appendix also introduces a circle model of localized competition, which has the structure that downstream firms are symmetrically distributed around a circle as in Salop (1979), while customer locations are uniformly distributed. In this case, each downstream firm again has market power over nearby customers, but is constrained only by the next nearest downstream firm. In each of these generalizations, the timing of decisions is the same as in our main model, except that there are an arbitrary number of downstream firms. Both models allow for \( m \geq 2 \) equally efficient upstream firms.

Our main results generalize readily to the hub-and-spokes model.\textsuperscript{19} With \( n > 2 \) downstream competitors, vertical integration combines with exclusive dealing to foreclose equally efficient upstream competition and raise downstream prices. There is, however, an additional result from the spokes model: the equilibrium upstream price under vertical inte-

\textsuperscript{17}Non-localized competition means in general that a consumer may have first-choice preference over downstream products, but no strong second-choice preference, or, alternatively, a consumer has a most-efficient supplier of the downstream product, but other suppliers are equally efficient. For example, consider a case in which a consumer can buy either from a single local supplier or over the Internet from more distant suppliers. Non-localized competition also applies naturally to markets with consumer switching costs.

\textsuperscript{18}This property is reminiscent of Chamberlinian monopolistic competition; individual firms have power over price while competing against “the market”. See also Hart (1985a, 1985b) and Perloff and Salop (1985).

\textsuperscript{19}A key observation for the extension of our results to the hub-and-spokes model of downstream oligopoly is that prices are strategic complements (Bulow, Geanakoplos, and Klemperer, 1985). Thus, an exclusive contract that raises the marginal input price to a downstream competitor has the benefit of encouraging other downstream rivals to raise their prices also. These infectious effects enable a vertically integrated cartel organizer to achieve higher downstream prices by bringing the entire downstream industry under exclusive contracts. The argument is related to Davidson and Deneckere’s (1985) analysis of incentives to form coalitions.
gration decreases in the number of downstream competitors. This suggests that market concentration in the downstream market can be important for the evaluation of the combined effects of vertical integration and exclusive contracts.

Our results also extend to the circle model of localized competition. In the circle model, the vertically integrated upstream firm only brings under exclusive contract its immediate downstream neighbors, while contracting efficiently with more distant downstream firms. Thus, in the case of four or more downstream firms, upstream competitors are excluded only from supplying the portion of the downstream market that is local to the integrated firm. Nevertheless, the combination of vertical integration and exclusive dealing has an anticompetitive effect in this local market segment.

Taken together, the hub-and-spokes model and the circle model indicate that the extent of upstream foreclosure and downstream cartelization depends on the nature (localized versus non-localized) of competition. The main complication in the more general models of bilateral oligopoly is the possibility that exclusive contracts have a cartelizing effect even under vertical separation. This possibility is eliminated with a parameter restriction that the upstream industry is not too concentrated compared to the downstream industry. The parameter restriction, which rules out the possibility of multiple equilibria, qualifies our conclusion that vertical integration is necessary for anticompetitive effects of exclusive dealing in the case of public contracting.

Private contracts

To keep the exposition of our results as simple as possible, we have maintained the assumption that bilateral requirements are public. In our earlier working paper (Chen and Riordan, 2003), however, we arrive at essentially the same conclusions under the alternative assumption that the contracts are private information. In business-to-business relationships especially, private contracting may be the more realistic assumption. The alternative assumption of private contracts has been an important one in the new vertical contracting literature (McAfee and Schwartz, 1994; O’Brien and Shaffer, 1992; Rey and Tirole, 2003; and Rey and Vergé, 2004). The privacy of contracts clearly is irrelevant for our upstream
monopoly case, because there are only two contracting parties and both necessarily observe offers and acceptances. The assumption potentially matters for the vertically integrated upstream duopoly case only if $D2$ were to reject $U1-D1$’s equilibrium offer. In this circumstance, $U1-D1$ must form some conjecture about a possible deviation contract that has offered by $U2$ and accepted by $D2$. The natural conjecture, however, is $r_2 = c$ because that would maximize bilateral profits.\(^{20}\) The distinction between public and private contracts, however, matters much more in the vertical separation case. In that case, if $U1$ makes an out-of-equilibrium offer to $D1$, then $D1$ must form a conjecture of what offer $U1$ has made to $D2$. The arbitrariness of these beliefs allows for multiple equilibria. The new vertical foreclosure literature has focussed on the selection of equilibria by postulating particular structures of out-of-equilibrium beliefs.

Our results under the alternative assumption of private bilateral contracts are as follows; see Chen and Riordan (2003) for details. First, Proposition 2 holds under private contracts if $U1-D1$ always believes that $r_2 = c$, which, as mentioned above, is the natural belief. Second, Proposition 3 holds under symmetry beliefs and the additional refinement $r_i \geq c$. Symmetry beliefs means that if $D1$ receives an out-of-equilibrium offer from $U1$, then $D1$ believes that $U1$ has offered the same $r_1$ to $D2$ and, furthermore, that $D2$ will accept the offer. While we have not ruled out equilibria with $r_i < c$, any such equilibrium would be Pareto dominated for the industry by an equilibrium with $r_2 = c$. More importantly, the proposition is sufficient for our conclusion that vertical integration is important for the cartelization of the downstream industry.\(^{21}\) Finally, these results also extend naturally to the hub-and-spokes and circle models. Therefore, our conclusions do not depend crucially on the assumption of public contracts. Furthermore, with private contracts, we do not need the parameter restriction on market structure to conclude that vertical integration is

\(^{20}\)The concern about out-equilibrium-beliefs can be avoided entirely in this case by interpreting $U2$ as competitive fringe that always stands ready to supply at $r_2 = c$.

\(^{21}\)The result also holds under passive beliefs and the strategy restriction $r_i \geq c$. Under passive beliefs, $Dj$ maintains the belief that $Di$ has accepted an equilibrium contract even after receiving an out-of-equilibrium offer. Without the strategy restriction, a passive belief equilibrium does not exist except when $c = 0$, for the similar reason as in McAfee and Schwartz (1995).
necessary for exclusive requirements contracts to have a cartelizing effect.

4. Discussion

Our analysis has revealed a relationship between vertical integration and exclusive dealing that has gone unnoticed in the economics literature. A vertically integrated firm may have a special ability and incentive to use exclusive requirements contracts to exclude equally efficient upstream competitors and effect a cartelization of the downstream industry. The vertically integrated firm convinces independent downstream rivals to accept exclusive supply contracts on supracompetitive terms with a carrot and stick. The carrot takes the familiar form of some direct compensation to the downstream independent firm for forgoing a competitive supply alternative. The stick is the credible threat of a price war if the competitor declines the exclusive deal. This price war threat is credible because, under the exclusive contract, the vertically integrated firm internalizes the opportunity cost of foregone input sales, and consequently has a clear incentive to price less aggressively in the downstream market. This opportunity cost would disappear if $D2$ were to contract with $U2$. The special advantage of a vertically integrated firm for cartelizing the industry is that the price war threat reduces the necessary size of the carrot.

We developed this point in the context of a "bidding market" on downstream competition, in which there is a separate winner-take-all competition to supply each customer (Klemperer, 2005). Additional important elements of the downstream market structure are that the good is produced to order, firms have heterogeneous costs for serving different customers, and demand for the good has some price sensitivity. Such features generally better describe business-to-business transactions than mass consumer markets. For example, markets for ready-mixed concrete used in large construction projects have these features. While we developed a specific model of a bidding market, we expect our main insight about vertical integration and exclusive dealing applies to other models of downstream competition. The key idea behind our analysis - that a vertically integrated firm

\footnote{Alternatively, the downstream products are differentiated and customers have heterogeneous preferences.}
who supplies a downstream rival under an exclusive contract also raises its own opportunity cost when it raises the supply price, thus effectively committing to softer competition - is very general.

In other models of downstream competition, however, performance comparisons of vertically-integrated and vertically-separated market structures may be less sharp. An important property of our model is that an optimal contract between an independent upstream firm and independent downstream firm sets the supply price equal to cost and redistributes profits with a lump sum transfer. This property arises from the assumption that firm heterogeneity is common knowledge at the time of downstream competition, i.e. the $D_i$ perfectly understand who has the advantage in any particular competition. A consequence of this property is that the input is supplied competitively in a non-integrated environment. This competitive pricing principle need not hold in other models of differentiated price competition in which bidders are less certain of their relative positions at the time of setting prices. For example, consider a duopoly in which downstream competitors have upward sloping reaction curves that intersect once, and assume that an increase in the input price shifts up a firm’s reaction curve at the point of intersection. In this case, an independent upstream and independent downstream firm may have an incentive to negotiate an exclusive contract with a supracompetitive supply price as a way to soften competition. Thus, exclusive contracts in a non-integrated environment may also have a partially cartelizing effect. Such a strategic commitment to softer competition is lost when a vertically integrated firm supplies only itself at cost, resulting in a more competitive downstream market compared to the non-integrated environment. The integrated firm nevertheless might still have an incentive negotiating exclusive contracts that raise the supply price to the independent downstream firm. It seems plausible that the end result is an even less competitive downstream market than vertical separation, assuming the opportunity cost effect of vertical integration cum exclusive dealing is sufficiently strong. Demonstrating this result formally, however, would require a different analysis for a more general class of models. Our bidding model is simpler to analyze because there is no strategic incentive to use exclusive contracts
to soften competition in the vertically-separated environment.\textsuperscript{23}

We have also shown that the cartelizing ability of the vertically integrated firm may be limited when downstream firms are heterogeneous and contracts cannot be contingent on uncertain market conditions. In particular a complete cartelization remains elusive when downstream monopoly prices vary with non-contractible market conditions. In such circumstances, the extent to which a vertically integrated supplier is able to cartelize the downstream industry depends on the degree of concentration in the downstream market and on the degree to which downstream competition is localized. We demonstrate this in the context of alternative spatial models of downstream competition that allow an arbitrary number of competitors, the hub-and-spokes model and the circle models, which otherwise have the similar "bidding market" characteristics as our main model.

Hart and Tirole (1990) made an important contribution to the vertical integration literature by showing that vertical integration enables an upstream monopolist to overcome a commitment problem when bilateral contracts are private, and achieve an \textit{ex post} monopoly outcome in the downstream market. Rey and Tirole (2003) felicitously refer to this result as “restoring” monopoly power. The essential logic is that a vertically integrated firm better internalizes the opportunity cost of cutting supply prices to downstream rivals. The same logic carries over if the upstream firm competes against inferior upstream rivals, although the ability to achieve a full monopoly outcome is constrained by potential competition from the less efficient suppliers. The Hart-Tirole-Rey theory does not explain an incentive for partial vertical integration if the upstream rivals are equally efficient. Our analyses show that such an incentive does exist if a vertically-integrated upstream firm has recourse to exclusive contracts. By charging a higher marginal supply price to downstream rivals, the vertically integrated supplier engineers a “more collusive” downstream outcome. The resulting increase in industry profits is shared among market participants \textit{via} lump sum transfers.\textsuperscript{24}

\textsuperscript{23}Interestingly though, this would not be an issue if requirements contracts were private as in Chen and Riordan (2003). Private contracts, however, complicate the analysis in other ways.

\textsuperscript{24}Alternatively, the upstream monopolist could solve the commitment problem by contracting with down-
market, Rey and Vergé (2004) demonstrate a similar problem of opportunism in a model of downstream price-setting, creating a similar role for vertical integration to restore monopoly power.

Aghion and Bolton (1987) made an important contribution to the literature on exclusive contracting by showing how penalty contracts could exclude an equally or more efficient entrant. Our analysis complements theirs by showing how a vertically integrated firm can use exclusive contracts to exclude an equally or more efficient firm who is already in the market. As suggested by the Chicago School, the exclusion of the upstream competitor is costly to the integrated firm, i.e. transfer payments are needed to gain the acquiescence of the downstream industry. But the necessary transfer payments are not so large as to make *ex post* cartelization unprofitable for the vertically integrated upstream firm. Interestingly, this cost approaches zero when the heterogeneity between downstream firms disappears: the vertically integrated firm relies on cutting its downstream prices as a (hidden) threat to persuade the independent downstream firms to accept the exclusive contract; this threat provides the most powerful incentive, and hence there is little need for explicit transfer payment, when the downstream producers become perfect substitute for each other.

We discuss two antitrust cases that illustrate the empirical relevance of our theory. One case is Kodak v. F.T.C. (1925). Kodak had a 90% market share for raw cinematic film that it supplied to downstream picture-makers. Kodak acquired capacity to enter the downstream industry, and reached essentially an exclusive-dealing agreement with picture-makers in which it agreed not to deploy the capacity if picture-makers would refrain from purchasing imported raw film. The Court found this agreement to be an illegal restraint of trade. Another case is TEKAL/ITALCEMENTI (A76), brought up by the Italian Antitrust Authority against Italcementi, the main cement manufacturer in Sardinia, Italy.25 Faced with lower-priced competition from imported cement, Italcementi acquired ten concrete

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25The discussion of this case is based on Italian Antitrust Authority Annual Report 1994, published on April 30, 1995. We thank Pierluigi Sabbatini of the Italian Antitrust Authority for directing our attention to this case.

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production facilities between April and June 1993, and began to sell its concrete at prices below variable cost, with the intention of dissuading the independent concrete producers from purchasing their cement from importers. It was then able to enter into contractual agreements with some main concrete purchasing companies that effectively excluded other concrete producers. The Italian Antitrust Authority ruled that the conduct of Italcementi was part of an overall plan to restrict access to the Sardinian cement market and constituted an abuse of dominant position, and it fined the company 3.75 billion lire.

While these two cases occurred in different times, countries, and industries, the strategic considerations involved in both of them are remarkably similar to those in our theory. In both cases, a vertically integrated upstream producer entered into exclusive contracts with independent downstream firms that excluded other upstream firms from market access. The independent downstream firms appeared to be willing to accept such arrangements because the integrated upstream producer used its downstream facilities to entice and discipline the independents: if the independents purchased inputs from the vertically integrated upstream producer, the vertically integrated downstream producer would compensate the independents by reducing or refraining from competition; otherwise it would aggressively cut prices. In other words, the credible threat of a price war encouraged downstream competitors to enter into an exclusive supply arrangement on supracompetitive terms - the opportunity cost effect identified in our model. As a result, the vertically integrated firm was able to exclude upstream competitors and likely also raised downstream prices. We also notice that the key features of our model are possibly present in the cases. In particular, for TEKAL/ITALCEMENTI (A76), the different downstream concrete producers likely had different shipping costs for consumers at different locations; downstream market condition was likely to be uncertain in that the location and the demand of a final customer might be unknown ex ante; and pricing contracts between a cement (upstream) producer and a

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26 Interestingly, there is a case similar to TEKAL/ITALCEMENTI (A76) in New Zealand, concerning a vertically integrated cement/concrete company, Fletcher Concrete and Infrastructure Limited. In 2002, the New Zealand Commerce Commission concluded that company’s conduct had the purpose and effect of excluding competition in the cement market and raising concrete prices, and issued a warning.
concrete (downstream) producer did not appear to be contingent on the locations of final consumers. Although the details of the two cases are different from our theoretical model, they do illustrate the empirical relevance of our argument that vertical integration raises heightened concerns about exclusive dealing and *vice versa*.

We close by discussing policy implications at a more general level. If our theory is to be useful for policies concerning vertical mergers and/or exclusive contracts, it must be supported by evidence on market structure. Our analysis suggests the following relevant evidence:

- Sole source requirements contracting is a normal industry practice or at least has some industry precedent. Otherwise, the theory might be judged as too speculative about post-merger industry conduct.

- Upstream price competition is “tough” before the merger or the use of exclusive contracts by a vertically integrated firm, as would be the case if the firms have similar capabilities and were not colluding tacitly (Sutton, 1991). Otherwise, there may be little to gain from cartelization *via* exclusive contracts, or the vertically-integrated firm might be unable to exclude a more efficient upstream competitor.

- The vertically-integrated firm is likely to have substantial excess capacity or can expand capacity easily. Otherwise, the integrated firm is unlikely to be able to supply other downstream firms on competitive terms.

- The downstream market is concentrated, and there are barriers to entry. Otherwise, the cartelization effect is small relative to the size of the market, or would be undone by new entry.²⁷

- Finally, evidence in favor of a plausible efficiency theory should be weighed against evidence in support of an anticompetitive effect (Riordan and Salop, 1995).²⁸

²⁷Market definition is a key issue when competition is localized. Sales to customer groups with few real alternatives may constitute a distinct product market.

²⁸For example, if the upstream competition were “soft”, as would be the case if the upstream firms colluded
Appendix

The Appendix contains proofs for the main results of Section 3.

Proof of Lemma 1: We organize our proof in two parts:

(1): If \( P_1^m \left( \frac{1}{2} \right) \geq r + \frac{1}{2} \tau \), then the proposed pricing strategies by \( D_1 \) and \( D_2 \) constitute a Nash equilibrium.

First consider the cases where \( x \leq \frac{1}{2} \). Notice that \( \tau x \leq \tau(1-x) \). From standard arguments in Bertrand competition, \( P_1(x, r) \) maximizes the joint profits of \( U1-D1 \) given \( D2 \)'s offer, \( D2 \)'s offer is optimal for \( D2 \) given \( P_1(x, r) \), and the consumer will select the firm with the lower cost, which is \( D1 \) here. The consumer will make the actual purchase if \( P_1(x, r) \leq v \).

Next consider the cases where \( x > \frac{1}{2} \). Notice that \( \tau x > \tau(1-x) \) in these cases. Notice also that, since \( P_1^m(x) - c - \tau x \) decreases in \( x \), we may possibly have \( P_1^m(x) < r + \tau x \) even though \( P_1^m \left( \frac{1}{2} \right) \geq r + \frac{1}{2} \tau \). We proceed with two possible situations:

(i) Suppose \( P_1^m(x) > r + \tau x \). At \( P_2(x, r) = \min \{ P_2^m(x, r), r + \tau x \} \), with the customer selecting \( D2 \), the expected profit of \( U1-D1 \) is \( [r - c] [1 - F(P_2(x, r))] \).

If \( D1 \) undercuts \( D2 \) so that it would be selected by the customer, the expected profit of \( U1-D1 \) is less than

\[
[r + \tau x - (c + \tau x)] [1 - F(r + \tau x)] \leq [r - c] [1 - F(P_2(x, r))].
\]

On the other hand, given \( D1 \)'s offer, it is optimal for \( D2 \) to charge \( P_2(x, r) \) and to be selected by the customer. Thus the proposed strategies constitute a Nash equilibrium.

(ii) Suppose instead \( P_1^m(x) \leq r + \tau x \). We have \( r + \tau (1-x) < r + \frac{1}{2} \tau \leq P_1^m \left( \frac{1}{2} \right) < P_1^m(x) \).

With the same logic as above, competition between \( D1 \) and \( D2 \) must drive the price down to \( P_1^m(x) \), and the consumer selects \( D2 \).

(2): If \((t, r)\) is an equilibrium contract, then indeed \( P_1^m \left( \frac{1}{2} \right) \geq r + \frac{1}{2} \tau \).

Suppose that, to the contrary, there is an equilibrium contract \((t, r)\) such that \( P_1^m \left( \frac{1}{2} \right) < r + \frac{1}{2} \tau \). We shall show that the expected industry profit is higher under an alternative expressly or tacitly, and if uniform pricing were the normal pre-merger industry practice, then the merger arguably might increase economic efficiency by eliminating a double markup.
contract \((t', r')\), or simply under \(r'\), where \(P^m_1(1/2) = r' + \frac{1}{2} \tau\). Since \(t\) and \(t'\) will be chosen such that the expected profits of \(D2\) are zero under the respective contracts, it follows that the expected profit for \(U1-D1\) must be higher under contract \((t', r')\) than under contract \((t, r)\), which produces a contradiction.

First consider the cases where \(x \leq 1/2\). Since \(\tau x \leq \tau (1 - x)\) and

\[
P^m_1(x) \leq P^m_1 \left( \frac{1}{2} \right) \leq r' + \tau (1 - x) < r + \tau (1 - x),
\]

the equilibrium price will be \(P_1(x, r) = P^m_1(x)\), under either \(r\) or \(r'\), and the customer will select \(D1\). Therefore for \(x \leq 1/2\), both contracts produce the same expected industry profits.

Now consider the cases where \(x > 1/2\). Then \(P^m_1(x) < r + \tau x\) from \(P^m_1 \left( \frac{1}{2} \right) < r + \frac{1}{2} \tau\) and the fact that \(P^m_1(x) - \tau x\) decreases in \(x\). Thus

\[
 r' + \tau (1 - x) < r' + \frac{1}{2} \tau = P^m_1 \left( \frac{1}{2} \right) < P^m_1(x) < r' + \tau x.
\]

Let \(\hat{x} > 1/2\) be such that either \(\hat{x}\) uniquely solves

\[
P^m_1(\hat{x}) = r + \tau (1 - \hat{x}),
\]

or \(\hat{x} = 1\) if \(P^m_1(1) < r\). Then for \(1/2 < x < \hat{x}\), \(P^m_1(x) < r + \tau (1 - x)\).

Hence, under \(r\), the equilibrium price will be \(P^m_1(x)\) but \(D1\) will be selected by the customer for \(1/2 < x < \hat{x}\); while under \(r'\) the equilibrium price will also be \(P^m_1(x)\) but \(D2\) will always be selected by the customer for \(1/2 < x \leq 1\). Therefore, for \(1/2 < x \leq 1\), industry profits will be higher under \(r'\) than under \(r\), since \(\tau (1 - x) < \tau x\).

Thus expected industry profits are higher under \(r'\) than under \(r\), contradicting that \((t, r)\) is an equilibrium contract. \(Q.E.D.\)

**Proof of Proposition 1:** We only need prove that \(P^m_1(0) - \tau < \hat{r} < P^m_1 \left( \frac{1}{2} \right) - \frac{1}{2} \tau\); everything else follows directly from Lemma 1 and from Assumption A2.

We first show that \(P^m_1(0) - \tau < \hat{r}\). Suppose to the contrary \(P^m_1(0) - \tau \geq \hat{r}\). Then, \(P^m_1(0) > \hat{r} + \tau x\) and \(P^m_1(0) > \hat{r} + \tau (1 - x)\), for all \(x \in (0, 1)\). We thus have

\[
P_1(x, \hat{r}) = \hat{r} + \tau (1 - x) < P^m_1(0) < P^m_1(x) \text{ for } 0 < x \leq \frac{1}{2},
\]

\[
P_2(x, \hat{r}) = \min \{ P^m_2(x, \hat{r}), \hat{r} + \tau x \} < P^m_1(0) < P^m_1(1 - x) \text{ for } \frac{1}{2} < x < 1.
\]
By raising \( \hat{r} \) slightly above \( P_1^m(0) - \tau \), both \( P_1(x, \hat{r}) \) and \( P_2(x, \hat{r}) \) will be closer to \( P_1^m(x) \) and \( P_1^m(1 - x) \), respectively, for all \( 0 < x < 1 \), which would lead to a higher expected industry profit than under \( \hat{r} \leq P_1^m(0) - \tau \). This implies that it cannot be optimal for \( U1 \) to offer \( \hat{r} \leq P_1^m(0) - \tau \); and therefore \( \hat{r} > P_1^m(0) - \tau \).

We next show that \( \hat{r} < P_1^m \left( \frac{1}{2} \right) - \frac{1}{2} \tau \). It suffices to show that \( \hat{r} \neq P_1^m \left( \frac{1}{2} \right) - \frac{1}{2} \tau \), since from Lemma 1 \( \hat{r} \leq P_1^m \left( \frac{1}{2} \right) - \frac{1}{2} \tau \). Now, from the proof of Lemma 1, if \( \hat{r} = P_1^m \left( \frac{1}{2} \right) - \frac{1}{2} \tau \), the equilibrium prices would be \( P_1(x, \hat{r}) = P_1^m(x) \) for \( x \leq \frac{1}{2} \) and

\[
P_2(x, \hat{r}) = \min \{ P_2^m(x, \hat{r}), \min \{ P_1^m(x), \hat{r} + \tau x \} \} > P_1^m(1 - x) \quad \text{for} \quad \frac{1}{2} < x \leq 1.
\]

That is, \( P_1(x, \hat{r}) \) is optimal for \( x \leq \frac{1}{2} \) while \( P_2(x, \hat{r}) \) is inefficiently too high for \( x > \frac{1}{2} \). A slight reduction in \( \hat{r} \) would reduce both \( P_1(x, \hat{r}) \) and \( P_2(x, \hat{r}) \) for \( x \) that is close to \( \frac{1}{2} \), causing a first-order increase in industry profits for those \( x \) that are to the right of \( \frac{1}{2} \) and a second-order decrease in industry profits for those \( x \) that are to the left of \( \frac{1}{2} \). Therefore, in equilibrium \( \hat{r} \neq P_1^m \left( \frac{1}{2} \right) - \frac{1}{2} \tau \). Q.E.D.

*Proof of Lemma 2:* First, when \( D2 \) contracts with \( U2 \) as its supplier, let \( \tilde{x} \) be such that

\[
c + \tau \tilde{x} = r_2 + \tau (1 - \tilde{x}),
\]

or \( \tilde{x} = \frac{r_2 - c}{2 \tau} + \frac{1}{2} \). Then \( \tilde{x} \) is the marginal customer for \( D1 \) and \( D2 \). In the downstream market competition, any customer with \( x \leq \tilde{x} \) will select \( D1 \) as the potential seller, and any consumer with \( x > \tilde{x} \) will select \( D2 \) as the potential seller. We can restrict our attention to \( \tilde{x} \in [0, 1] \), since it is not optimal for \( U2 \) to offer some \( r_2 \) that results in \( \tilde{x} \) outside of this interval.

Second, given any \( r_2 \) and for any \( x > \tilde{x} \), the equilibrium price for \( D2 \) must be

\[
\tilde{P}_2(x, r_2) = \min \{ c + \tau x, P_2^m(x, r_2) \},
\]

But since

\[
P_2^m(x, r_2) \geq P_2^m(x, c) = P_1^m(1 - x) > P_1^m(0) > c + \tau x,
\]

we have

\[
\tilde{P}_2(x, r_2) = c + \tau x \quad \text{for} \quad x > \tilde{x}.
\]
The joint profits of $U2$ and $D2$ are

$$
\Pi_2(r_2) = \int_{\bar{x}}^{1} [(c + \tau x) - (c + \tau (1 - x)) [1 - F(c + \tau x)] dx

= \int_{\bar{x}}^{1} \tau(2x - 1) [1 - F(c + \tau x)] dx, 
$$

where recall that $\bar{x} = \frac{r_2-c}{2\tau} + \frac{1}{2}$. Thus,

$$
\Pi_2(r_2) = -\tau(2\bar{x} - 1) [1 - F(c + \tau\bar{x})] \frac{1}{2\tau}, 
$$

which is positive if $\bar{x} < \frac{1}{2}$, or equivalently if $r_2 < c$; and is negative if $\bar{x} > \frac{1}{2}$, or equivalently if $r_2 > c$. Thus $\tilde{r}_2 = c$. Q.E.D.

**Proof of Proposition 2:** A Pareto optimal contract between $U1-D1$ and $D2$ (i.e. a contract that maximizes the profit of $U1-D1$ given a profit constraint for $D2$) maximizes $\Pi(r_1)$, while a Pareto optimal contract between $U2$ and $D2$ has $r_2 = c$ by Lemma 2. Therefore, we can restrict attention to contracts with $r_1 = \hat{r}$ and $r_2 = c$. Given this restriction, and given the expected profits determined according to Lemma 1, competition at Stage 1 is reduced to an asymmetric procurement auction in which $U1-D1$ and $D2$ bid $t_1$ and $t_2$ for the exclusive right to supply $D2$ at prices $\hat{r}$ and $c$ respectively. Since $\Pi(\hat{r}) > \Pi(c)$ by Proposition 1, standard reasoning implies that in (the unique refined) equilibrium $U2$ bids $t_2 = 0$ and $U1-D1$ wins with $t_1^*$ defined in equation (7). Since $r_1 = \hat{r}$, the downstream equilibrium outcome is the same as under upstream monopoly by Proposition 1. It remains to show that $t_1^* < 0$. Notice that when $r_1$ increases, $P_2(x,r_1)$ is either unchanged when $P_2(x,r_1) = P_1^m(x)$, or increases otherwise. In addition, $P_2(x,r_1) - \tau(1-x) - r_1$ is non-increasing in $r_1$, and is strictly decreasing in $r_1$ if $P_2(x,r_1) = P_1^m(x)$. Therefore,

$$
t_1^* < \int_{\frac{1}{2}}^{1} [P_2(x,c) - \tau(1-x) - c] [1 - F(P_2(x,c))] dx - \int_{\frac{1}{2}}^{1} \tau(2x - 1) [1 - F(c + \tau x)] dx = 0, 
$$

where the last equality is a consequence of Assumption $A2$; that is, when $r_1 = c$, $D1$’s willingness to sell at cost always constrains $D2$’s market power. Q.E.D.

**Proof of Proposition 3:**

We organize the proof in two steps, noticing that we need only be concerned with contracts in which $r_j \geq c$. In step 1, we show that any $r_i > c$ cannot occur in equilibrium. We then
show in step 2 that there exists an equilibrium where \((t_j^*, r_j^*) = (0, c)\), for \(j = 1, 2\). Since step 1 implies that at any possible equilibrium \(r_j^* = c\), and hence \(t_j^* = 0\), combining step 1 and step 2 completes our proof.

**Step 1. There can be no equilibrium where \(r_i > c\) for any \(i\).**

Suppose to the contrary that there is some equilibrium where \(r_i > c\) for at least one \(i\). Without loss of generality, suppose that \(r_1 > c\) and \(r_2 \geq c\). There are two possible cases to consider.

Case 1: \(r_1\) and \(r_2\) are offered by the two different upstream firms. Without loss of generality, suppose \(r_1\) is from \(U1\) and \(r_2\) is from \(U2\). The marginal consumer between \(D1\) and \(D2\), \(\tilde{x}\), satisfies

\[
\tilde{x}(r_1, r_2) \equiv \frac{r_2 - r_1}{2\tau} + \frac{1}{2}.
\]

It is easy to see that \(\tilde{x}(r_1, r_2) \in (0, 1)\), since otherwise one of the upstream-downstream pair would have zero joint profit; and by a contract with \(r_i = c\) the pair can have a positive expected profit. So suppose that \(\tilde{x}(r_1, r_2) \in (0, 1)\). The joint profits of \(U1-D1\) under \(r_1\), given \(r_2\), is

\[
\Pi_{11}(r_1, r_2) = \int_0^{\tilde{x}} \left[ \min \left\{ P^m(x, r_1), r_2 + \tau (1 - x) \right\} - (c + \tau x) \right] \cdot \left[ 1 - F \left( \min \left\{ P^m(x, r_1), r_2 + \tau (1 - x) \right\} \right) \right] dx.
\]

A reduction of \(r_1\) to \(c\) would increase \(\tilde{x}\); in addition, it would either increase or have no effect on

\[
\left[ \min \left\{ P^m(x, r_1), r_2 + \tau (1 - x) \right\} - (c + \tau x) \right] \cdot \left[ 1 - F \left( \min \left\{ P^m(x, r_1), r_2 + \tau (1 - x) \right\} \right) \right].
\]

Therefore a reduction of \(r_1\) to \(c\) would increase the joint profits of \(U1-D1\), or \(\Pi_{11}(c, r_2) > \Pi_{11}(r_1, r_2)\) for \(r_1 > c\). This shows that there can be no equilibrium where the downstream firms are supplied by the two separate upstream firms and at least one downstream firm contracts to receive the input at a unit price above \(c\).

Case 2: \(r_1\) and \(r_2\) are offered by a single upstream firm, say, \(U1\). Let \(t_1\) and \(t_2\) be the transfer payments from \(D1\) and \(D2\) to \(U1\) at the proposed possible equilibrium. Let \(\Pi(r_1, r_2)\)
denote the joint profits of $U_1$, $D_1$, and $D_2$ when $D_1$ and $D_2$ contract with $U_1$ under $r_1$ and $r_2$. Then
\[
\Pi(r_1, r_2) = \int_0^{\tilde{x}(r_1, r_2)} [P(x, r_1, r_2) - (c + \tau x)] [1 - F(P(x, r_1, r_2))] \, dx \\
+ \int_{\tilde{x}(r_1, r_2)}^1 [P(1 - x, r_2, r_1) - (c + \tau(1 - x))] [1 - F(P(1 - x, r_2, r_1))] \, dx
\]
\[
= \Pi_{11}(r_1, r_2) + \Pi_{12}(r_1, r_2) = \Pi_{21}(r_1, r_2) + \Pi_{22}(r_1, r_2).
\]

Given $(t_1, r_1)$ accepted by $D_1$, an offer from $U_2$ to $D_2$ with $r_2' = c$ will produce the following joint profit between $U_2$-$D_2$:
\[
\Pi_{22}(r_1, c) = \int_{\max\{\tilde{x}(r_1, c), 0\}}^1 \min\{\tilde{x}(r_1, c), 0\} [\min(P^m_r(r_1, c), r_1 + \tau x) - (c + \tau(1 - x))] \\
\cdot [1 - F(\min(P^m_r(r_1, c), r_1 + \tau x))] \, dx.
\]
Thus, in order to prevent $D_2$ from deviation, we must have
\[-t_2 \geq \Pi_{22}(r_1, c) \geq \Pi_{22}(r_1, r_2) = \Pi_{12}(r_1, r_2),\]
where we notice that the $U_2$-$D_2$ pair would maximize its joint profit by setting $r_2 = c$, given any $r_1 \geq c$. On the other hand, given $(t_2, r_2)$ accepted by $D_2$, an offer from $U_2$ to $D_1$ with $r_1' = c$ will produce the following joint profit between $U_2$-$D_1$:
\[
\Pi_{21}(c, r_2) = \int_0^{\min(\tilde{x}(c, r_2), 1)} \min(P^m_r(c, r_1 + \tau x) - (c + \tau(1 - x))] \\
\cdot [1 - F(\min(P^m_r(c, r_1 + \tau (1 - x)))] \, dx.
\]
Thus, in order to prevent $D_1$ from deviating, we must have
\[-t_1 \geq \Pi_{21}(c, r_2) > \Pi_{21}(r_1, r_2) = \Pi_{11}(r_1, r_2),\]
since $r_1 > c$. Therefore for the proposed contracts to be part of an equilibrium, it is necessary that
\[-(t_1 + t_2) \geq \Pi_{22}(r_1, c) + \Pi_{21}(c, r_2) > \Pi_{22}(r_1, r_2) + \Pi_{21}(r_1, r_2) = \Pi(r_1, r_2),\]
which means that $U_1$ will have to receive a negative profit. Therefore there can be no equilibrium where the downstream firms are supplied by a single upstream firm and at least one downstream firm contracts to receive the input at a unit price above $c$.  

We have thus shown that there can be no equilibrium where \( r_i > c \) for any \( i \).

Step 2. There exists an equilibrium in which \((t_{ij}, r_{ij}) = (0, c)\) for \( i, j = 1, 2 \), \( D1 \) accepts the contract offered by \( U1 \), and \( D2 \) accepts the contract offered by \( U2 \).

We need to show that an upstream firm cannot benefit from deviating offers to either one downstream firm or to both downstream firms.

First, given the contract offered by \( U2 \) and that \( D2 \) accepts \( U2 \)’s contract, \( r_{11} = c \) maximizes the joint profits of \( U1-D1 \), and it is optimal for \( U1 \) to set \( t_{11} = 0 \) given that \((t_{21}, r_{21}) = (0, c)\), and optimal for \( D1 \) to accept \( U1 \)’s offer. Similarly, given the contract offered by \( U1 \) and that \( D1 \) accepts \( U1 \)’s contract, \( r_{22} = c \) maximizes the joint profits of \( U2-D2 \), and it is optimal for \( U2 \) to set \( t_{22} = 0 \) given that \((t_{12}, r_{12}) = (0, c)\), and optimal for \( D2 \) to accept \( U2 \)’s offer.

Next, suppose that an upstream firm, say \( U1 \), offers both \( D1 \) and \( D2 \) a deviating contract that involves some \( r_j > c \) and is accepted. With the same logic in Case 2 of Step 1 above, either one of the \( D_j \) will receive a higher payoff by staying with \( U2 \) under \( r = c \), or \( U1 \) will receive a negative payoff. Thus any such deviation cannot be successful. \textit{Q.E.D.}

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