Innovation in Vertically Related Markets

by

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Abstract

We examine the impact of vertical industry structure on upstream process innovation. We find that vertical integration (VI) generally enhances innovation under downstream Cournot competition, but can diminish innovation under downstream Bertrand competition. We also find that under Bertrand competition, VI can increase innovation when the direct incentives for innovation are limited, but can reduce innovation when the direct incentives are pronounced.

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1 Introduction.

The literature has analyzed in considerable detail how the horizontal structure of an industry – the number of firms, in particular – affects incentives for process innovation.[1] The literature has devoted much less attention to the corresponding issue of how the vertical structure of an industry affects innovation. A thorough understanding of the impact of vertical industry structure on innovation is important for antitrust authorities as they review proposed vertical mergers. Such understanding is of particular importance in light of recent calls for vertical divestiture in network industries in order to limit incentives for vertically-integrated suppliers of bottleneck inputs to disadvantage retail rivals.[2] Although vertical divestiture may limit the incentives of an upstream supplier to advantage its downstream affiliate and disadvantage rivals, vertical divestiture may also reduce innovation incentives unduly.[3]

The purpose of this research is to determine whether upstream process innovation is more pronounced when the monopoly supplier of an essential input also competes in the downstream (retail) market or when no such vertical integration is present. Under both vertical industry structures, we analyze downstream competition between two suppliers of a homogeneous retail product. To illustrate, the essential input might be access to a ubiquitous telecommunications network and the retail product might be basic local telephone service. The upstream supplier (U) can devote non-contractible, costly effort to reducing its upstream unit cost of production. Through its design of the essential input, U also can influence the relative costs of the two downstream suppliers (D1 and D2) without altering their average unit cost of production.[4] A fraction of any upstream cost reduction that U achieves accrues to

[1] See, for example, Arrow (1962), Loury (1979), Reinganum (1982), and Aghion et al. (2005). Gilbert (2006) provides a recent review of the literature.

[2] See Crew et al. (2005), for example. Such disadvantaging of retail rivals can increase their operating costs and thereby reduce their competitive strength (Salop and Scheffman, 1983).

[3] Crandall and Sidak (2002), for example, suggest that this may be the case.

[4] Thus, U can reduce upstream production costs but cannot reduce aggregate downstream costs under any vertical industry structure.

other producers. This spillover might reflect unavoidable leakage of technological knowledge to a competitive fringe of upstream producers, for example. Alternatively, it might reflect a regulatory policy that reduces the price of the essential input as upstream production costs decline. D1 and D2 engage either in Bertrand competition or in Cournot competition after upstream production costs, the input design, and the input price have been determined.

We derive two main conclusions. First, the impact of vertical industry structure on upstream process innovation varies with the form of downstream competition. Vertical integration (VI) generally increases innovation under downstream Cournot competition but can reduce innovation under Bertrand competition. Second, under Bertrand competition, the impact of VI on innovation varies with the prevailing direct incentives for innovation. VI can increase innovation in the presence of weak direct incentives (e.g., weak patent policies or regulated input prices that closely track realized upstream costs), but can reduce innovation in the presence of strong direct incentives for innovation.

VI generally increases innovation under Cournot competition for two reasons. First, an input design that increases D2's cost while reducing D1's cost by the same amount leaves industry output unchanged under Cournot competition. Because the disadvantaging of the retail rival that arises under VI does not reduce industry output, VI does not diminish U's incentive to increase the upstream profit margin by reducing upstream costs. Second, an upstream cost reduction endows D1 with a strategic competitive advantage under Cournot competition. This advantage arises because D1 internalizes the entire cost reduction while D2 secures only the portion of the reduction that is passed along in the form of a lower input price. For both these reasons, VI generally increases innovation under Cournot competition.

In contrast, VI can reduce innovation under Bertrand competition for two reasons. First, U's disadvantaging of D2 under VI reduces industry output under Bertrand competition,[5] and thereby reduces U's incentive to increase its profit margin. Second, a reduction in upstream costs does not endow D1 with the same strategic advantage that it does under

[5] This is the case because the equilibrium price reflects D2's unit cost of production under Bertrand competition.

Cournot competition. This is the case because D1 recognizes that if it attracts some of D2's customers, D2 will reduce its purchase of the essential input, thereby reducing U's profit. D1's resulting reluctance to react aggressively to an upstream cost reduction offsets any strategic advantage that the reduction might otherwise provide for D1. The failure of an upstream cost reduction to endow D1 with a strategic downstream advantage, coupled with reduced industry output, ensure that VI can reduce innovation under Bertrand competition.

The prevailing direct incentives for innovation affect the impact of VI on upstream process innovation under Bertrand competition for the following reasons. When direct incentives for innovation are limited, the input price closely tracks the realized upstream cost. Consequently, U perceives little gain from reducing the upstream cost under vertical separation (VS). Under VI, though, U values an upstream cost reduction because it increases D1's profit by expanding its output.[6] Thus, VI increases innovation when direct incentives for innovation are limited.

In contrast, in the presence of strong direct incentives for innovation, U will have substantial incentive to increase its profit margin by reducing upstream production costs under VS. VI can reduce this incentive in part by reducing industry output as D2's costs, and thus the equilibrium retail price, rise. Reduced industry output implies reduced sales of the input, which limits U's financial return from increasing its profit margin by reducing its upstream production cost.

These findings highlight the importance of considering the nature of the prevailing retail competition (and not simply the number of retail competitors) when assessing the merits of a proposed vertical merger. The findings also imply that while VI can substitute for weak direct incentives for innovation, it can limit innovation in the presence of strong direct incentives. VI is particularly likely to stifle innovation when the upstream supplier has considerable latitude to influence the relative costs of downstream suppliers and when the

[6] In equilibrium, D1 serves the entire industry output, and industry output expands because the upstream cost reduction leads to a lower input price, which reduces the equilibrium retail price. The higher output level increases the number of units of output on which D1 earns a positive profit margin.

suppliers are engaged in fierce price competition.

We develop and present our findings as follows. Section 2 describes the central elements of our basic model. Sections 3 and 4 examine equilibrium outcomes under Cournot competition and under Bertrand competition, respectively. Section 5 reviews our findings, considers extensions of our analysis, and suggests directions for future research. The proofs of all formal findings are presented in the Appendix.

Before proceeding, we briefly review the works that are most closely related to our own. Buehler and Schmutzler (2005) provide a complementary analysis of the impact of VI on downstream, rather than upstream, process innovation. The authors demonstrate that VI can increase incentives for innovation because the output expansion that results from a downstream cost reduction generates increased upstream profit that is internalized by a vertically-integrated downstream innovator.[7] Choi et al. (2003) prove that such incentives can induce downstream process innovation in excess of socially optimal levels.[8] Buehler et al. (2004) show that VI often increases quality-enhancing investment in a regulated network industry because an integrated firm internalizes both the increased upstream and downstream profit that results from the quality improvement.[9] Brocas (2003) analyzes a model with upstream duopoly in which both upstream producers can pursue and license cost-reducing innovation. She finds that if downstream producers can readily employ the technologies of either upstream producer, the competition among upstream producers can lead to low license fees and limited incentives for upstream innovation.

2 Elements of the Model.

We consider a setting with two downstream producers of a homogeneous product (D1 and D2) and one upstream producer (U). Under vertical separation (VS), all three producers

[7] Also see Farrell and Katz (2000).

[8] Bannerjee and Lin (2003) demonstrate that downstream innovation can increase the demand for an essential input and thereby increase the price of the input, which raises the costs of retail rivals.

[9] Buehler et al. (2006) demonstrate that vertical divestiture cannot both reduce the retail price and increase the quality of an essential input supplied by a regulated monopolist.

are independent entities, each of which acts to maximize its own profit. Under vertical integration (VI), D2 is an independent, profit-maximizing entity while U and D1 form a single enterprise (denoted U-D1) that acts to maximize the joint profit of U and D1.

Under both vertical market structures, exactly one unit of an essential upstream product ("the input") is required to produce each unit of the downstream (retail) product. Di's additional unit cost of producing the retail product after securing the essential input is denoted $c_i(\cdot)$ for $i = 1, 2$. U's unit cost of producing the input is denoted c^u.[10] Initially, c^u is $c_h > 0$. U can reduce c^u by $\alpha \geq 0$ units at personal cost $K(\alpha)$, where $K(\cdot)$ is a strictly increasing, strictly convex function of α, with $\lim_{\alpha \to 0} K(\alpha) = 0$, $\lim_{\alpha \to c_h} K(\alpha) = \infty$, and $\lim_{\alpha \to 0} K'(\alpha) = 0$.

U retains the fraction $t \in [0, 1]$ of any upstream cost reduction that it achieves. Formally, when U secures cost reduction α, the unit price at which U sells the input is:

$$w = c_h - \alpha + t\alpha = c_h - [1 - t]\alpha. \tag{1}$$

This sharing of the benefits of U's upstream process innovation admits several interpretations. For instance, U may face potential competition from fringe producers of the input who initially also have unit cost c^u. In this setting, $1 - t$ is the fraction of U's realized process innovation that accrues to fringe producers via technological spillover. In order to continue to supply the input to independent downstream producers, U must reduce the price at which it sells the input to the level of the unit cost of potential competitors, which is $c_h - [1 - t]\alpha$.

Alternatively, U might be a regulated monopoly supplier of the input. In this setting, the regulator requires U to pass on to downstream producers the fraction $1 - t$ of any upstream cost reduction it achieves. U retains the fraction t of its realized innovation in the form of an input price that exceeds the upstream unit cost of production $(c_h - \alpha)$ by $t\alpha$.

In addition to determining the level of its upstream production cost, U can influence the relative costs of the two downstream producers. In practice, an upstream supplier might

[10] We abstract from fixed costs of production for expositional simplicity. Fixed costs could easily be added to the model without affecting our findings. Indeed, the presence of an upstream monopoly could reflect large fixed costs.

influence relative downstream costs by determining the extent to which the essential input meets the idiosyncratic needs (or is otherwise compatible with the production processes) of the downstream producers.[11] We model U's design of the input as the choice of a parameter $L \in [0,1]$. U's choice of L does not affect either its upstream unit production cost or the sum of the downstream unit production costs of D1 and D2. However, lower levels of L reduce D1's unit production cost ($c_1(\cdot)$) while increasing D2's corresponding cost ($c_2(\cdot)$).[12] As assumption 1 indicates, we presume that D1 has the lowest downstream production cost when the input design is chosen to minimize D1's cost.[13]

Assumption 1. $c_1'(L) > 0$ and $c_1'(L) + c_2'(L) = 0$ for all $L \in [0,1]$. Also, $c_2(0) - c_1(0) > 0$.

Assumption 1 implies that if D2 has the lowest downstream production cost when the input design is chosen to minimize D2's cost (so $c_1(1) > c_2(1)$), then there is input design $L \in (0,1)$ that ensures D1 and D2 have the same downstream unit production cost. In contrast, if D1 always has the lowest downstream production cost (so $c_2(1) \geq c_1(1)$), then the input design that minimizes the difference between the downstream unit costs of D1 and D2 is $L = 1$. We will denote by L^e the value of $L \in [0,1]$ that minimizes the difference between these downstream unit costs.[14]

The timing in the model is as follows. First, the industry structure (VS or VI) and the direct incentives for innovation (t) are determined.[15] Second, U chooses the extent of upstream cost reduction (α) and the input design (L). Third, the realized upstream unit cost ($c_h - \alpha$) is observed and the unit input price (w) is determined. Fourth, D1 and D2

[11] This aspect of our analysis resembles Gilbert and Riordan's (2007) model of technological tying in which an upstream producer can design an input so as to differentially reduce the quality of the retail products of downstream rivals.

[12] For instance, suppose U can locate its facility at any point $L \in [0,1]$ without affecting its marginal cost of production, c^u. Also suppose D1 is located at 0 and D2 is located 1, and both suppliers must travel to U's facility to purchase the input. Then D1 and D2 will incur transportation costs L and $1 - L$, respectively.

[13] Assumption 1 is maintained throughout the ensuing analysis. The concluding section considers the different qualitative conclusions that could arise under alternative assumptions.

[14] Thus, $L^e \in (0,1)$ and $c_1(L^e) = c_2(L^e)$ when $c_1(1) > c_2(1)$, whereas $L^e = 1$ when $c_2(1) \geq c_1(1)$.

[15] We take the vertical industry structure and direct incentives to be exogenous in order to focus on how they affect innovation. The concluding section discusses endogenous direct incentives.

choose their strategies simultaneously and independently. D1 and D2 choose output levels q_1 and q_2, respectively, under Cournot competition. D1 and D2 choose retail prices p_1 and p_2, respectively, under Bertrand competition. Fifth, all realized equilibrium retail demand is satisfied. Under Cournot competition, the equilibrium retail price (p) equates consumer demand to the sum of the outputs of D1 and D2. Under Bertrand competition, all retail demand is served by the firm that sets the lowest price.[16] We shall use $r \in \{B, C\}$ to denote the prevailing competition regime, where "B" denotes Bertrand competition and "C" denotes Cournot competition. The retail demand function is denoted $Q(p)$. The corresponding inverse demand function is denoted $P(Q)$.

3 Cournot Competition.

We begin by considering the setting in which D1 and D2 engage in Cournot competition after the industry structure (VS or VI), the input design (L), the upstream cost reduction (α), and the input price (w) are determined. Under VS, Di chooses output q_i to:

$$Maximize \;\; q_i \left[P\left(q_i + q_{-i}\right) - \left(w + c_i(\cdot)\right) \right], \tag{2}$$

for $i, -i = 1, 2$ and $i \neq -i$. Under VI, D2 chooses q_2 to maximize the expression in (2) for $i = 2$. Also, q_1 is chosen under VI to maximize the sum of upstream operating profit and the downstream profit of D1. Formally, q_1 is chosen to:

$$Maximize \;\; [w - c^u][q_1 + q_2] + q_1 \left[P\left(q_1 + q_2\right) - \left(w + c_1(\cdot)\right) \right].^{17} \tag{3}$$

Notice that (3) can be rewritten as:

[16] When D1 and D2 set the same retail price, in equilibrium the firm with the lower unit cost serves all retail demand.

[17] For expositional simplicity, we restrict attention to settings in which D1 and D2 both produce strictly positive output in equilibrium. This will be the case, for example, when consumer demand for the homogenous retail product is sufficiently pronouned and when the production costs of D1 and D2 are sufficiently low and sufficiently similar for all values of $L \in [0, 1]$. Notice that U-D1 does not have unfettered ability to simply monopolize the downstream industry by foreclosing D2. U-D1 is compelled to sell the input to D2 at price $w = c_h - [1 - t]\alpha$ (by the fringe of competitive upstream suppliers or by regulatory fiat, for example).

$$Maximize \quad [w - c^u]\, q_2 + q_1 \left[P\left(q_1 + q_2\right) - \left(c^u + c_1\left(\cdot\right)\right)\right]. \tag{4}$$

As expression (4) reveals, D1's effective unit cost of production under VI with downstream Cournot competition is $c^u + c_1\left(\cdot\right)$.

Let $\bar{c} \equiv c_1(L) + c_2(L)$, and let c_Σ^j denote the sum of the effective unit costs of production for D1 and D2 under vertical industry structure $j \in \{S, L\}$, where "S" denotes vertical separation and "I" denotes vertical integration here and throughout the ensuing analysis. Equation (1) implies:

$$c_\Sigma^S \;=\; 2w + \bar{c} \;=\; 2c_h - 2[1-t]\alpha + \bar{c}, \quad \text{and} \tag{5}$$

$$c_\Sigma^I \;=\; c^u + w + \bar{c} \;=\; 2c_h - [2-t]\alpha + \bar{c}. \tag{6}$$

Summing the necessary conditions for the solutions to the maximization problems in (2) and (3) reveals that equilibrium industry output under vertical industry structure $j \in \{S, L\}$, denoted $Q^*\left(c_\Sigma^j\right)$, is given by:

$$2\,P\left(Q^*\left(c_\Sigma^j\right)\right) + [Q^*\left(c_\Sigma^j\right)]P'\left(Q^*\left(c_\Sigma^j\right)\right) \;=\; c_\Sigma^j. \tag{7}$$

Equation (7) implies that equilibrium industry output varies with the sum of the effective unit production costs of D1 and D2. From equations (5) and (6), this sum is independent of L under both VS and VI. Because industry output and U's upstream profit margin are both independent of L under VS, U is indifferent among all feasible input designs under VS in the presence of downstream Cournot competition. Under VI, U prefers the input design that advantages D1 to the maximum extent possible ($L = 0$). This design increases D1's downstream profit (by increasing D1's output and reducing its costs) without reducing U's upstream profit (since the choice of L does not affect total industry output).

These observations are recorded formally in Lemma 1. The lemma refers to L^S, which is the input design that maximizes U's profit under VS, and to L^I, which is the input design that maximizes U-D1's profit under VI.

Lemma 1. *$L^I = 0$, while L^S can be any $L \in [0,1]$ under downstream Cournot competition.*[18]

Anticipating the input designs identified in Lemma 1, U chooses the level of upstream cost reduction (α) under VS to maximize the difference between upstream operating profit and innovation costs. Upstream operating profit is the product of the upstream profit margin ($w - c^u$) and industry output. Formally, since the upstream profit margin is $t\alpha$ (from equation (1)), U chooses α to maximize:

$$\Pi^{SC}(\alpha) \;=\; [w - c^u] Q^*\left(c_\Sigma^S\right) - K(\alpha) \;=\; t\alpha Q^*\left(c_\Sigma^S\right) - K(\alpha). \tag{8}$$

Under VI, U-D1 can be viewed as choosing α to maximize the sum of upstream profit ($[w - c^u]Q(\cdot) - K(\cdot)$) and D1's downstream profit ($q_1[P(\cdot) - w - c_1(\cdot)]$), where D1 is regarded as purchasing the input from U at unit price $w = c_h - [1-t]\alpha$. Let $q_i^*(\alpha)$ denote Di's equilibrium output under VI with downstream Cournot competition, given upstream cost reduction α. Then U-D1 can be viewed as choosing α to maximize:

$$\Pi^{IC}(\alpha) \;=\; t\alpha Q^*(c_\Sigma^I) - K(\alpha) + q_1^*(\alpha)\left[P\left(Q^*\left(c_\Sigma^I\right)\right) - \left(c_h - [1-t]\alpha + c_1^I\right)\right], \tag{9}$$

where $c_i^j \equiv c_i(L^j)$ for $i = 1, 2$ and $j \in \{S, I\}$. $\Pi^{SC}(\alpha)$ and $\Pi^{IC}(\alpha)$ are assumed to be strictly concave functions. We will denote by α^{jC} the value of α that maximizes $\Pi^{jC}(\alpha)$ for $j \in \{S, I\}$. Proposition 1 compares the equilibrium levels of upstream process innovation under VS and VI in the presence of downstream Cournot competition.

Proposition 1. *Upstream process innovation is greater under VI than under VS in the presence of downstream Cournot competition (i.e., $\alpha^{IC} > \alpha^{SC}$) for all $t \in [0,1]$ when $|P''(Q)|$ is sufficiently small for all $Q \geq 0$.*

To understand the conclusion in Proposition 1 and the corresponding conclusions to follow, it is important to recognize the different ways in which innovation affects profit. Recall that profit is the product of the number of units sold and the profit margin earned on

[18] U's preferred level of upstream cost reduction (α) depends only on total industry output under VS. Because this output does not vary with L under Cournot competition, U's choice of α will be unique under VS even though L^S is not unique.

each unit. Upstream process innovation can increase profit by increasing the number of units sold (by reducing the relevant sales price) and/or by increasing the relevant profit margin (by reducing costs).

The input design (L) is chosen under VI to increase D2's costs and reduce D1's costs. Because this favoring of D1 does not change the sum of the downstream unit production costs, it does not affect industry output under Cournot competition. Therefore, VI does not reduce U's incentive to increase its profit margin (by reducing its upstream production cost).

In addition, VI confers a strategic advantage upon D1 that increases with the level of upstream cost reduction under Cournot competition. Notice from equation (6) that D1 perceives its unit cost of production to be $c^u + c_1(0)$ (rather than $w + c_1(0)$) when it is integrated with U. Consequently, a \$1 reduction in c^u translates into a full \$1 reduction in D1's effective marginal cost. The same \$1 reduction in c^u only reduces D2's marginal cost by $\$(1 - t)$, which is the rate at which the input price declines as the upstream production cost declines. Therefore, the larger is the upstream cost reduction that U achieves, the more pronounced is the effective cost advantage that D1 obtains over D2, and thus the greater is the increase in D1's profit.

A complete assessment of the impact of innovation on profit requires an analysis of the rate at which relevant output levels and profit margins change as the upstream unit cost declines. A comparison of the rates at which outputs change with c^u under different vertical industry structures is complicated by the fact that equilibrium output levels differ across industry structures. However, when the curvature of the retail demand function is sufficiently limited (i.e., when $|P''(Q)|$ is sufficiently small), the equilibrium outputs will vary with changes in c^u at similar rates across industry structures. Thus, it is convenient to focus on settings where the retail demand function has limited curvature when comparing innovation levels across vertical industry structures.[19]

[19] The requirement that $|P''(Q)|$ be sufficiently small for all $Q \geq 0$ is overly restrictive. The key requirement is that the slope of the demand function at the equilibrium price under VI be sufficiently close to the corresponding slope at the equilibrum price under VS.

The greater upstream cost reduction under VI identified in Proposition 1 supports a lower input price and thus a lower retail price. D1's full internalization of the upstream cost reduction under VI in the presence of Cournot competition also supports expanded retail output and thus a lower retail price. These two effects together lead to a higher level of consumer surplus under VI than under VS, as Corollary 1 reports. The corollary refers to $Z^{jr}(t)$, which is the equilibrium level of consumer surplus under vertical structure $j \in \{S, L\}$ and competition regime $r \in \{B, C\}$, when the prevailing level of direct incentives is t.

Corollary 1. *Consumer surplus is higher under VI than under VS in the presence of downstream Cournot competition (i.e., $Z^{IC}(t) > Z^{SC}(t)$) for all $t \in [0, 1)$ when $|P''(Q)|$ is sufficiently small for all $Q \geq 0$.*[20]

Proposition 1 reveals that under downstream Cournot competition, VI typically increases upstream process innovation even though the integrated producer acts to disadvantage its retail rival. Furthermore, VI enhances innovation whether the prevailing direct incentives for innovation are pronounced or limited. The analysis in section 4 reveals that these conclusions are sensitive to the nature of downstream competition.

4 Bertrand Competition.

Now consider the setting in which D1 and D2 engage in Bertrand competition after the vertical industry structure, the input design, the upstream cost reduction, and the input price are determined. To characterize U's preferred input design (L) in this setting, notice that U's operating profit under VS is:

$$[w - c^u] \; Q(\max\{w + c_1(L), \; w + c_1(L)\}). \tag{10}$$

The expression in (10) is the product of U's upstream profit margin ($w - c^u$) and downstream industry output. Thus, given a positive upstream profit margin, U will choose L to maximize

[20] It is also readily shown that total surplus is higher under VI than under VS under the conditions specified in Corollary 1.

industry output. As expression (10) indicates, the equilibrium retail price under Bertrand competition is the maximum of the unit production costs of D1 and D2. Therefore, to maximize industry output, U will choose L to minimize the maximum downstream unit production cost. Assumption 1 implies that this maximum unit cost is minimized when the difference between the two unit costs is minimized. Consequently, U's profit-maximizing choice of L under VS (denoted L^S) is L^e.

To characterize U's corresponding choice of input design under VI, notice that the equilibrium retail price under VI is again:

$$\max\{w + c_1(L),\, w + c_2(L)\}. \tag{11}$$

This is the case because the integrated supplier (U-D1) faces both a physical cost and an opportunity cost for each unit of retail output that it produces and sells. The physical cost is the sum of the upstream and downstream unit production costs, $c^u + c_1(L^I)$, where, recall, L^I denotes U-D1's profit-maximizing input design under VI. The opportunity cost is the upstream profit margin $(w - c^u)$ that U-D1 foregoes when D1, rather than D2, delivers a unit of retail output, and so D2 purchases one less unit of the essential input from U. The sum of the physical unit cost and the opportunity unit cost of expanding retail output is:

$$c^u + c_1(L^I) + w - c^u \;=\; w + c_1(L^I). \tag{12}$$

Expression (12) implies that U-D1 acts as if its unit cost of production is $w + c_1(L^I)$ under VI, and so the equilibrium retail price is as specified in expression (11).[21]

Expression (11) implies that under VI with downstream Bertrand competition, U-D1 will choose L to maximize:

$$[w - c^u]Q\left(\max\{w + c_1(L), w + c_2(L)\}\right) + \max\{[c_2(L) - c_1(L)]Q\left(w + c_2(L)\right), 0\} \tag{13}$$

[21] See Chen (2001) for further explanation and discussion of this observation. Recall that D1 does not perceive a corresponding opportunity cost of output expansion under Cournot competition because D1 regards D2's output as fixed when it selects its own output level.

$$= \begin{cases} [w + c_2(L) - (c^u + c_1(L))]Q(w + c_2(L)) & if \quad c_2(L) - c_1(L) \geq 0 \\ [w - c^u]Q(w + c_1(L)) & if \quad c_2(L) - c_1(L) < 0. \end{cases} \tag{14}$$

The two terms in expression (13) are U's upstream operating profit and D1's downstream profit, respectively. The second term in expression (13) and the terms in expression (14) reflect the fact that D1 serves all retail demand at price $w+c_2(L)$ if and only if $c_1(L) < c_2(L)$. (Recall expression (11).)

To characterize L^I, it is convenient to assume that D2 always imposes meaningful price discipline on D1. Formally, we assume throughout the ensuing analysis that U-D1's profit would increase if the retail price were increased above its equilibrium level under Bertrand competition. From equation (14), this will be the case if assumption 2 holds.

Assumption 2. $[p-c^u-c_1(L)]\ Q(p)$ is strictly increasing in $p = w+c_2(L)$ for all $w \in [0,\ c_h]$ and for all $L \in [0, 1]$.[22]

Assumption 2 ensures that $L^I = 0$ because this input design increases D2's unit cost (and thus the equilibrium retail price) to the maximum extent possible.

Lemma 2 summarizes the profit-maximizing input design under Bertrand competition.

Lemma 2. $L^I = 0$ *and* $L^S = L^e \in (0, 1]$ *under downstream Bertrand competition.*

To characterize the impact of vertical industry structure on upstream process innovation under Bertrand competition, it follows from expression (10) that U will choose α under VS to maximize:

$$\Pi^{SB}(\alpha) \ = \ [w - c^u]Q(w + c_2^S) - K(\alpha) \ = \ t\alpha Q(c_h - \alpha[1 - t] + c_2^S) - K(\alpha). \tag{15}$$

The second equality in equation (15) follows from equation (1), since $c^u = c_h - \alpha$.

Under VI, U-D1's profit margin given retail price $p = w + c_2^I$ is:

[22] Assumption 2 implies that U-D1's profit increases as $c_2(L)$ increases. An increase in $c_2(L)$ has two effects: it increases U-D1's profit margin $(w+c_2(L)-(c^u+c_1(L)))$ and reduces equilibrium output. Thus, assumption 2 requires the increase in profit due to the higher profit margin to outweigh the corresponding reduction in profit due to reduced output. This will be the case, for example, if equilibrium output is sufficiently large and/or if retail demand is sufficiently price inelastic. Assumption 2 simplifies the ensuing discussion considerably without altering the basic insights developed below.

$$p - c^u - c_1^I = w + c_2^I - c^u - c_1^I = w - c^u + c_2^I - c_1^I. \tag{16}$$

Therefore, since $w - c^u = t\alpha$ from equation (1), U-D1 will choose α under VI to maximize:

$$\Pi^{IB}(\alpha) = [t\alpha + c_2^I - c_1^I] Q(c_h - [1-t]\alpha + c_2^I) - K(\alpha). \tag{17}$$

$\Pi^{SB}(\alpha)$ and $\Pi^{IB}(\alpha)$ are assumed to be strictly concave functions. We will denote by α^{jB} the value of α that maximizes $\Pi^{jB}(\alpha)$, for $j \in \{S, I\}$. Proposition 2 provides sufficient conditions for α^{IB} to exceed α^{SB}, i.e., for greater process innovation to arise under VI than under VS.

Proposition 2. *Upstream cost reduction is greater under VI than under VS in the presence of downstream Bertrand competition (i.e., $\alpha^{IB} > \alpha^{SB}$) if one of the following holds: (i) t is sufficiently close to zero; (ii) $t < \frac{1}{2}$ and $Q''(p)$ is sufficiently small for all $p \geq 0$; or (iii) U's ability to affect relative downstream costs is sufficiently limited (so $c_2(0) - c_2(1)$ is sufficiently small) and $t < 1$.*

The higher level of innovation under VI reported in Proposition 2 stems in part from the larger relevant profit margin that prevails under VI than under VS. U's profit margin under VS is $w - c^u$. U-D1's profit margin under VI is as specified in equation (16). Notice that $c_2^I - c_1^I = c_2(0) - c_1(0) > 0$ from Lemma 2 and assumption 1. Therefore, for a given input price and upstream production cost, U-D1's profit margin under VI exceeds U's profit margin under VS.

When direct incentives for innovation are (nearly) absent (so $t \to 0$), the input price declines by (nearly) the full amount of any upstream cost reduction. Consequently, upstream process innovation does not increase the profit margin under either VS or VI. However, the reduction in the input price increases industry output at the rate $|Q'(w + c_2^S)|$ under VS and at the rate $|Q'(w + c_2^I)|$ under VI. The expanded output increases the profit of the integrated supplier under VI because its profit margin $(c_2^I - c_1^I)$ is positive. The increased output under VS does not increase U's profit (much) because its profit margin $(w - c^u)$ is (nearly) zero. Therefore, on balance, when direct incentives for innovation are (nearly) absent, upstream process innovation will be greater under VI than under VS.

For corresponding reasons, upstream cost reduction continues to be more pronounced under VI than under VS when the direct incentives for innovation are sufficiently limited (so $t < 1/2$) and when the rate at which industry output increases as the input price declines is not too much higher under VS than under VI. Because the equilibrium retail price is higher under VI than under VS in the presence of Bertrand competition, the rate of increase in industry output will be at least as high under VI as under VS if $Q''(p)$ is sufficiently small (i.e., $Q''(p) \lessapprox 0$).

When downstream costs are largely insensitive to the input design (so $c_2(0) - c_2(1) \to 0$), U's choice of $L^I = 0$ will not result in a significant reduction in industry output under VI. Consequently, the primary effect of upstream process innovation is to increase industry output (by reducing the input price and thus the equilibrium retail price). The increased output generates a larger increase in profit under VI than under VS because of the larger profit margin under VI. (Recall equation (16).) Therefore, upstream process innovation will be more pronounced under VI than under VS when D2's downstream cost is largely unaffected by the input design.

As Corollary 2 reports, the combination of greater upstream cost reduction and little effective disadvantaging of D2 under VI when $c_2(0) - c_2(1) \to 0$ results in a lower equilibrium price, and thus a higher level of consumer surplus, under VI than under VS.

Corollary 2. *Consumer surplus is higher under VI than under VS in the presence of downstream Bertrand competition if U's ability to affect relative downstream costs is sufficiently limited (i.e.,* $Z^{IB}(t) > Z^{SB}(t)$ *for all* $t \in [0, 1)$ *if* $c_2(0) - c_2(1)$ *is sufficiently small).*[23]

In contrast, upstream cost reduction will be more pronounced and consumer surplus will be higher under VS than under VI when the direct incentives for innovation are sufficiently strong (so $t \to 1$). When the input price does not decline (much) as the upstream cost

[23] It is readily shown that total surplus (the sum of consumer surplus and profit) is also higher under VI than under VS when $c_2(0) - c_2(1)$ is sufficiently small.

declines, an upstream cost reduction will not reduce the retail price or increase industry output (much). Consequently, the predominant effect of an upstream cost reduction is to increase the relevant profit margin. The rate at which profit increases as the profit margin increases is directly proportional to the level of industry output. Industry output is larger under VS than under VI because U designs the input to disadvantage D2 (and, thereby raise the equilibrium retail price) under VI. Therefore, the incentives for upstream process innovation are more pronounced under VS than under VI when t is sufficiently close to 1. This logic explains the conclusions in Proposition 3 and Corollary 3.

Proposition 3. *Upstream cost reduction is more pronounced under VS than under VI in the presence of downstream Bertrand competition when the direct incentives for innovation are sufficiently pronounced (i.e., $\alpha^{SB} > \alpha^{IB}$ when t is sufficiently close to 1).*

Corollary 3. *Consumer surplus is higher under VS than under VI in the presence of downstream Bertrand competition (i.e., $Z^{SB}(t) > Z^{IB}(t)$) when t is sufficiently close to 1.*

Together, Propositions 2 and 3 and their corollaries reveal that although VI can increase upstream process innovation and consumer surplus under downstream Bertrand competition when direct incentives for innovation are limited, VI can reduce innovation and consumer welfare when direct incentives are pronounced.

5 Conclusions.

The findings in sections 3 and 4 support two primary conclusions. First, the impact of vertical industry structure on the incentives for upstream process innovation vary with the form of downstream competition. While vertical integration (VI) tends to promote process innovation in the presence of Cournot competition, VI may reduce innovation under Bertrand competition. Second, under Bertrand competition, the prevailing direct incentives for innovation affect the impact of VI on innovation. VI will increase upstream process innovation when the direct incentives are sufficiently limited, but will reduce innovation when the direct incentives are sufficiently pronounced.

The form of downstream competition affects the impact of VI on innovation for two main reasons. First, the advantaging of D1 and the disadvantaging of D2 that takes place under VI reduces equilibrium industry output under Bertrand competition but not under Cournot competition. The reduced output under Bertrand competition reduces U-D1's incentive to undertake the process innovation that increases the upstream profit margin. Second, process innovation conveys a more pronounced strategic advantage upon D1 under Cournot competition than under Bertrand competition. This is the case because D1 effectively internalizes the entire upstream cost reduction under Cournot competition, but not under Bertrand competition.

The conclusion that the impact of VI on innovation varies with the prevailing direct incentives under Bertrand competition is also readily explained. When the direct incentives are pronounced (so t is close to 1), the predominant effect of VI is to reduce industry output by increasing D2's production costs. The reduced output reduces U-D1's incentive to increase its profit margin. In contrast, when the direct incentives are limited (so t is close to 0), an upstream cost reduction generates little increase in the profit margin. Consequently, the incentives for upstream cost reduction are limited under VS. VI can enhance these incentives by allowing U to benefit from the increase in D1's profit that arises as industry output expands in response to reduced upstream costs.

Our findings emphasize the need to consider both the nature of downstream competition and the prevailing direct incentives for innovation when assessing the impact of VI on upstream innovation. VI tends to enhance innovation when downstream competition is relatively intense (i.e., Bertrand) but direct incentives are limited or when downstream competition is less intense (i.e., Cournot). In contrast, VI can reduce upstream innovation when downstream competition is intense and direct incentives for innovation are pronounced.

Additional research is required to determine the extent to which the conclusions derived in our simple model persist more generally. It can be shown that our key qualitative conclusions continue to hold in the setting where U can, at personal cost $k(\theta)$, choose the probability

(θ) with which the upstream unit cost is reduced from c_h to $c_h - \Delta$, for fixed $\Delta \in (0, c_h]$. Thus, the key conclusions are robust to some forms of stochastic upstream innovation. More general forms of stochastic innovation remain to be explored.

Alternative forms of competition also merit investigation. When the retail products of the downstream suppliers are heterogeneous, for example, the equilibrium retail price under price competition may be less sensitive to D2's costs than it is when retail products are homogeneous. Consequently, U's favoring of D1 under VI may have a reduced impact on equilibrium output, and thus on incentives for upstream cost reduction under price competition.[24] Of course, the extent and nature of competition among multiple active upstream suppliers also will influence incentives for innovation.[25]

Alternative effects of the input design (L) on downstream costs also warrant consideration. If lower values of L increase industry costs ($c_1(\cdot) + c_2(\cdot)$) as they raise D2's costs, then the disadvantaging of D2 that arises under VI can serve to increase the retail price under Cournot competition as it does under Bertrand competition. Consequently, the impact of vertical industry structure on upstream innovation may become less clear cut under Cournot competition.[26]

Our analysis has taken the magnitude of direct incentives to be exogenous and identical across vertical industry structures. It can be shown, though, that some of our main qualitative conclusions continue to hold when the direct incentives are chosen under each industry structure to maximize consumer surplus. In particular, under suitable regularity

[24] In the extreme case where the retail demands for the products of D1 and D2 are independent, U will perceive only a cost (reduced demand by D2 for the input) and no benefit (i.e., no strategic advantage for D1 in its downstream competition with D2) from raising D2's unit cost of production.

[25] Brocas (2003) analyzes this issue in a related, but distinct, model.

[26] Our analysis is readily extended to consider the case in which D2 has the lowest downstream unit production cost for all input designs (so $c_2(0) < c_1(0)$). The analysis under Cournot competition is unchanged by this alternative assumption. Under Bertrand competition, D2 will always serve the entire retail demand and U will set $L = 0$ under both VS and VI. This input design reduces D1's unit cost and thus the equilibrium retail price to the maximum extent possible, and thereby maximizes U's sales of the input to D2. Because D1 never operates and so always secures zero profit, the incentives for innovation under VS and VI are identical when $c_2(0) < c_1(0)$ in the presence of downstream Bertrand competition.

conditions,[27] the optimal level of direct incentives is higher under VI than under VS in the presence of downstream Cournot competition.[28] The higher level of direct incentives, coupled with the relatively strong incentives under VI for any given level of direct incentives (recall Proposition 1), ensure that innovation and consumer surplus are higher under VI than under VS in the presence of downstream Cournot competition when the direct incentives are chosen to maximize consumer surplus under both vertical industry structures.[29]

Future research should consider both upstream and downstream innovation. The overall impact of vertical industry structure on industry innovation likely will depend in part on the extent to which upstream and downstream innovation are substitutes or complements. Product innovation, in addition to process innovation, also merits detailed analysis.[30] Information sharing, collaborative research and development, and endogenous industry structure[31] also are relevant issues that warrant formal modeling.

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[27] The conditions are $|P''(Q)|$ and $|K'''(\alpha)|$ are sufficiently small for all $Q \geq 0$ and $\alpha \geq 0$. These conditions ensure that the curvature of the demand function is sufficiently limited and the cost of upstream innovation is sufficiently close to being quadratic.

[28] These more pronounced direct incentives arise in part because direct incentives are less costly to provide under VI in the presence of Cournot competition than in the presence of Bertrand competition. This is the case because D1 internalizes the entire realized upstream cost reduction under Cournot competition, regardless of the fraction of the reduction that U is permitted to retain. Consequently, D1 continues to act relatively aggressively as t increases under Cournot competition, and so the equilibrium retail price remains relatively low even when U retains a sizable portion of any cost reduction it achieves. Therefore, consumers are best served by relatively strong direct incentives for innovation under VI.

[29] A corresponding ranking of innovation under VS and VI is more problematic in the presence of downstream Bertrand competition. Recall from Proposition 2 that innovation tends to be greater under VI than under VS for fixed, small values of t. However, the level of direct incentives that maximizes consumer surplus is lower under VI than under VS in the presence of downstream Bertrand competition when the aforementioned regularity conditions hold. These reduced direct incentives for innovation under VI can offset the tendency for a higher level of innovation under VI (for fixed t) in the presence of downstream Bertrand competition.

[30] See Greenstein and Ramey (1998), Farrell and Katz (2000), Buehler et al. (2006), Chen and Schwartz (2007), and Gilbert and Riordan (2007), for example.

[31] See, for example, Gupta and Loulou (1998), Choi and Yi (2000), Buehler and Schmutzler (2005), Sadonís and Faulí-Oller (2006).

Appendix

Lemma A1. *Under VI in the presence of downstream Cournot competition: (i) $q_1^{*\prime}(\alpha) > 0$; and (ii) $q_2^{*\prime}(\alpha) \leq 0$ when $t > 1/2$ if $|P''(Q^*)|$ is sufficiently small.*

Proof. Differentiating (7) implies:

$$Q^{*\prime}\left(c_\Sigma^j\right) = \frac{1}{3P'\left(Q^*\left(c_\Sigma^j\right)\right) + Q^*\left(c_\Sigma^j\right) P''\left(Q^*\left(c_\Sigma^j\right)\right)} < 0. \tag{A1}$$

Denote the equilibrium outputs of D1 and D2 under VI in the presence of downstream Cournot competition by $q_1^*(\tilde{c}_1, \tilde{c}_2)$ and $q_2^*(\tilde{c}_1, \tilde{c}_2)$, respectively, where

$$\tilde{c}_1 \equiv c_h - \alpha + c_1^I \quad \text{and} \quad \tilde{c}_2 \equiv c_h - [1-t]\alpha + c_2^I \tag{A2}$$

are the unit costs of D1 and D2, respectively. Solving the problems identified in (2) and (3) and using (A2), q_1^* and q_2^* satisfy:

$$P(Q^*) - \tilde{c}_1 + q_1^*(\tilde{c}_1, \tilde{c}_2) P'(Q^*) = 0, \quad \text{and} \tag{A3}$$

$$P(Q^*) - \tilde{c}_2 + q_2^*(\tilde{c}_1, \tilde{c}_2) P'(Q^*) = 0. \tag{A4}$$

Recall that a firm's equilibrium output is decreasing in its own marginal cost and increasing in its rival's marginal cost under Cournot competition. Formally:

$$\frac{\partial q_i^*(\tilde{c}_1, \tilde{c}_2)}{\partial \tilde{c}_i} < 0 \quad \text{and} \quad \frac{\partial q_i^*(\tilde{c}_1, \tilde{c}_2)}{\partial \tilde{c}_j} > 0, \text{ for } i \neq j. \tag{A5}$$

From (A2), $\tilde{c}_2 - \tilde{c}_1 = t\alpha + c_2^I - c_1^I > t\alpha$, from assumption 1. Therefore:

$$q_1^*(\tilde{c}_1, \tilde{c}_2) > q_2^*(\tilde{c}_1, \tilde{c}_2). \tag{A6}$$

Now write $q_i^*(\tilde{c}_1, \tilde{c}_2)$ as $q_i^*(\alpha)$. Then, differentiating (A3) and (A4) with respect to α, and using (5) and (A2) provides:

$$-[2-t]P'(Q^*)Q^{*\prime}\left(c_\Sigma^I\right) + 1 + q_1'(\alpha)P'(Q^*) - [2-t]q_1P''(Q^*)Q^{*\prime}\left(c_\Sigma^I\right) = 0; \quad \text{and} \tag{A7}$$

$$-[2-t]P'(Q^*)Q^{*\prime}\left(c_\Sigma^I\right) + 1 - t + q_2'(\alpha)P'(Q^*) - [2-t]q_2P''(Q^*)Q^{*\prime}\left(c_\Sigma^I\right) = 0. \tag{A8}$$

Rearranging terms in (A7) and (A8) provides:

$$q_1^{*\prime}(\alpha) = \frac{[2-t]\left[P'(Q^*) + q_1^*P''(Q^*)\right]Q^{*\prime}\left(c_\Sigma^I\right) - 1}{P'(Q^*)}, \quad \text{and} \tag{A9}$$

$$q_2^{*\prime}(\alpha) = \frac{[2-t]\left[P'(Q^*) + q_1^*P''(Q^*)\right]Q^{*\prime}\left(c_\Sigma^I\right) - [1-t]}{P'(Q^*)}. \tag{A10}$$

From (6):

$$Q^{*\prime}(\alpha) = Q^{*\prime}\left(c_\Sigma^I\right)\frac{dc_\Sigma^I}{d\alpha} = -[2-t]Q^{*\prime}\left(c_\Sigma^I\right) > 0. \quad \text{(A11)}$$

The inequality in (A11) follows from (A1). Also $Q^{*\prime}(\alpha) = q_1^{*\prime}(\alpha) + q_2^{*\prime}(\alpha)$. Therefore, (A11) implies $q_i^{*\prime}(\alpha) > 0$ for $i = 1$ and/or $i = 2$.

If $q_1^{*\prime}(\alpha) \le 0$, then $q_2^{*\prime}(\alpha) > 0$. Consequently, from (A10):

$$\begin{aligned}
&[2-t]\left[P'(Q^*) + q_1^* P''(Q^*)\right] Q^{*\prime}\left(c_\Sigma^I\right) - [1-t] < 0 \\
\Rightarrow \quad & [2-t]\left[P'(Q^*) + q_1^* P''(Q^*)\right] Q^{*\prime}\left(c_\Sigma^I\right) - 1 < 0, \\
\Rightarrow \quad & q_1^{*\prime}(\alpha) = \frac{[2-t]\left[P'(Q^*) + q_1^* P''(Q^*)\right] Q^{*\prime}\left(c_\Sigma^I\right) - 1}{P'(Q^*)} \; > \; 0. \qquad \text{(A12)}
\end{aligned}$$

The contradiction implied by (A12) ensures $q_1^{*\prime}(\alpha) > 0$.

From (A10) and (A1):

$$\begin{aligned}
q_2^{*\prime}(\alpha) &= \frac{[2-t]\left[P'(Q^*) + q_1^* P''(Q^*)\right] Q^{*\prime}\left(c_\Sigma^I\right) - [1-t]}{P'(Q^*)} \\
&= \frac{1}{P'(Q^*)} \frac{[2-t]\left[P'(Q^*) + q_1 P''(Q^*)\right] - [1-t]\left[3P'(Q^*) + Q^* P''(Q^*)\right]}{3P'(Q^*) + Q^* P''(Q^*)} \\
&= \frac{1}{P'(Q^*)} \frac{[2t-1]P'(Q^*) + \left[(2-t)\, q_1^* - (1-t)\, Q^*\right] P''(Q^*)}{3P'(Q^*) + Q^* P''(Q^*)} \\
&\to \frac{[2t-1]}{3P'(Q^*) + Q^* P''(Q^*)} < 0 \text{ for } t > 1/2, \text{ as } P''(Q^*) \to 0, \qquad \text{(A13)}
\end{aligned}$$

where $\frac{1}{3P'(Q^*)+Q^*P''(Q^*)} < 0$ from (A1). ■

Proof of Proposition 1. The proof follows from the presumed concavity of the profit functions by showing that $\Pi^{IC\prime}(\alpha) > \Pi^{SC\prime}(\alpha)$ under the specified conditions.

From (8), U's profit under VS given equilibrium industry output $Q^*\left(c_\Sigma^S\right)$ is:

$$\Pi^{SC}(\alpha) \; = \; t\alpha Q^*\left(c_\Sigma^S\right) - K(\alpha). \qquad \text{(A14)}$$

(5) and (A14) imply that U's optimal choice of α under VS in the presence of downstream Cournot competition is determined by:

$$\Pi^{SC\prime}(\alpha) \; = \; tQ^*\left(c_\Sigma^S\right) - 2t[1-t]\alpha Q^{*\prime}\left(c_\Sigma^S\right) - K'(\alpha) = 0. \qquad \text{(A15)}$$

From (9), U-D1's profit under VI in the presence of downstream Cournot competition is:

$$\Pi^{IC}(\alpha) = t\alpha Q^*(c_\Sigma^I) - K(\alpha) + q_1^*(\alpha)\left[P(Q^*(c_\Sigma^I)) - \left(c_h - [1-t]\alpha + c_1^I\right)\right]. \qquad \text{(A16)}$$

For given t, U chooses α to maximize $\Pi^{IC}(\alpha)$. Therefore, since $\frac{dc_\Sigma^I}{d\alpha} = -[2-t]$ from (6),

$\alpha^I(t)$ under downstream Cournot competition is the value of α that solves:

$$\Pi^{IC\prime}(\alpha) = tQ^*\left(c_\Sigma^I\right) - t[2-t]\alpha Q^{*\prime}\left(c_\Sigma^I\right) + q_1^{*\prime}(\alpha)\left[P\left(Q^*\left(c_\Sigma^I\right)\right) - \left(c_h - [1-t]\alpha + c_1^I\right)\right]$$
$$+\, q_1^*(\alpha)\left[-P'(Q^*)\,Q^{*\prime}\left(c_\Sigma^I\right)[2-t] + 1 - t\right] - K'(\alpha) = 0. \qquad \text{(A17)}$$

From (A15) and (A17):

$$\Pi^{IC\prime}(\alpha) \;-\; \Pi^{SC\prime}(\alpha) \;=\; t\left[Q^*\left(c_\Sigma^I\right) - Q^*\left(c_\Sigma^S\right)\right] + t\alpha[2(1-t)\,Q^{*\prime}\left(c_\Sigma^S\right)$$
$$-(2-t)Q^{*\prime}\left(c_\Sigma^I\right)] + q_1^{*\prime}(\alpha)\left[P\left(Q^*\left(c_\Sigma^I\right)\right) - \left(c_h - [1-t]\alpha + c_1^I\right)\right]$$
$$+\, q_1^*(\alpha)\left[-P'(Q^*)\,Q^{*\prime}\left(c_\Sigma^I\right)(2-t) + 1 - t\right]. \qquad \text{(A18)}$$

Case 1. $t \leq 1/2$. (A1) implies that $P'(Q^*)\,Q^{*\prime}\left(c_\Sigma^I\right) \to 1/3$ as $P''(Q) \to 0$. Therefore, as $P''(Q) \to 0$:

$$-P'(Q^*)\,Q^{*\prime}\left(c_\Sigma^I\right)[2-t] + 1 - t \;\to\; \frac{-[2-t]}{3} + 1 - t \;=\; \frac{1-2t}{3} \geq 0. \qquad \text{(A19)}$$

(A2) implies:

$$P\left(Q^*\left(c_\Sigma^I\right)\right) - \left(c_h - [1-t]\alpha + c_1^I\right) = P(\cdot) - \widetilde{c}_1 - t\alpha > c_2^I - c_1^I > 0. \qquad \text{(A20)}$$

The first inequality in (A20) holds because:

$$P(\cdot) - [c_h - \alpha + t\alpha + c_2] = P(\cdot) - [w + c_2] > 0. \qquad \text{(A21)}$$

The inequality in (A21) follows from the maintained assumption that both firms produce strictly positive output in equilibrium, which implies that D2's profit margin must be positive.

(A20), Lemma A1, and assumption 1 imply:

$$q_1^{*\prime}(\alpha)\left[P\left(Q^*\left(c_\Sigma^I\right)\right) - \left(c_h - [1-t]\alpha + c_1^I\right)\right] \geq q_1^{*\prime}(\alpha)\left[c_2^I - c_1^I\right] > 0. \qquad \text{(A22)}$$

Furthermore, $c_\Sigma^S \geq c_\Sigma^I$ from (5) and (6), and so:

$$t\left[Q^*\left(c_\Sigma^I\right) - Q^*\left(c_\Sigma^S\right)\right] \geq 0. \qquad \text{(A23)}$$

In addition, as $P''(Q) \to 0$, $P'\left(Q^*\left(c_\Sigma^S\right)\right) - P'\left(Q^*\left(c_\Sigma^I\right)\right) \to 0$. Consequently, from (A1):

$$Q^{*\prime}\left(c_\Sigma^S\right) - Q^{*\prime}\left(c_\Sigma^I\right) \to \frac{1}{3P'\left(Q^*\left(c_\Sigma^S\right)\right)} - \frac{1}{3P'\left(Q^*\left(c_\Sigma^I\right)\right)} \to 0 \quad \text{as } P''(Q) \to 0. \qquad \text{(A24)}$$

(A24) implies:

$$2[1-t]Q^{*\prime}\left(c_\Sigma^S\right) - [2-t]Q^{*\prime}\left(c_\Sigma^I\right) = 2[1-t]Q^{*\prime}\left(c_\Sigma^S\right) - 2[1-t]Q^{*\prime}\left(c_\Sigma^I\right) - tQ^{*\prime}\left(c_\Sigma^I\right)$$
$$\to \quad -\,tQ^{*\prime}\left(c_\Sigma^I\right) \geq 0 \quad \text{when } P''(Q) \to 0. \qquad \text{(A25)}$$

(A18), (A19), (A22), (A23), and (A25) imply $\Pi^{IC\prime}(\alpha) - \Pi^{SC\prime}(\alpha) > 0$ when $t \leq 1/2$.

<u>Case 2</u>. $t > 1/2$. The mean value theorem, (5), and (6) imply that for some $\xi \in \left(c_\Sigma^I, c_\Sigma^S\right)$:

$$t\left[Q^*\left(c_\Sigma^I\right) - Q^*\left(c_\Sigma^S\right)\right] = tQ^{*\prime}(\xi)\left[c_\Sigma^S - c_\Sigma^I\right] = -t\left[t\alpha Q^{*\prime}(\xi)\right]. \tag{A26}$$

Also, $q_2^{*\prime}(\alpha) \leq 0$ when $t > 1/2$ from Lemma A1. Therefore $q_1^{*\prime}(\alpha) \geq Q^{*\prime}(\alpha)$, and so:

$$\begin{aligned} q_1^{*\prime}(\alpha)\left[P\left(Q^*\left(c_\Sigma^I\right)\right) - \left(c_h - [1-t]\alpha + c_1^I\right)\right] &= q_1^{*\prime}(\alpha)\left[-t\alpha + P(\cdot) - \widetilde{c}_1\right] \\ \geq\ Q^{*\prime}(\alpha)\left[-t\alpha + P(\cdot) - \widetilde{c}_1\right] &= Q^{*\prime}\left(c_\Sigma^I\right)\frac{dc_\Sigma^I}{d\alpha}\left[-t\alpha + P(\cdot) - \widetilde{c}_1\right] \\ =\ -[2-t]Q^{*\prime}\left(c_\Sigma^I\right)\left[-t\alpha - q_1^* P'(Q^*)\right]. & \end{aligned} \tag{A27}$$

The first equality in (A27) follows from (A20). The last equality in (A27) follows from (6) and (A3).

(A3) and (A20) imply:

$$-q_1^* P'(Q^*) = P - \tilde{c}_1 > t\alpha. \tag{A28}$$

(A18), (A26), (A20), and (A28) imply that for some $\xi \in \left(c_\Sigma^I, c_\Sigma^S\right)$:

$$\begin{aligned} \Pi^{IC\prime}(\alpha)\ &-\ \Pi^{SC\prime}(\alpha) \\ &= -t\alpha t Q^{*\prime}(\xi) + t\alpha[2(1-t)Q^{*\prime}\left(c_\Sigma^S\right) - (2-t)Q^{*\prime}\left(c_\Sigma^I\right)] + q_1^*(\alpha)[1-t] \\ &\quad + q_1^{*\prime}(\alpha)\left[-t\alpha - q_1^* P'(Q^*)\right] - q_1^*(\alpha) P'\left(Q^*\left(c_\Sigma^I\right)\right) Q^{*\prime}\left(c_\Sigma^I\right)[2-t] \\ &\geq -t\alpha t Q^{*\prime}(\xi) + t\alpha[2(1-t)Q^{*\prime}\left(c_\Sigma^S\right) - (2-t)Q^{*\prime}\left(c_\Sigma^I\right)] + q_1^*(\alpha)[1-t] \\ &\quad - [2-t]Q^{*\prime}\left(c_\Sigma^I\right)\left[-t\alpha - q_1^*(\alpha)P'(Q^*)\right] - q_1^*(\alpha) P'\left(Q^*\left(c_\Sigma^I\right)\right) Q^{*\prime}\left(c_\Sigma^I\right)[2-t] \qquad \text{(A29)} \\ &= -t\alpha t Q^{*\prime}(\xi) + t\alpha 2[1-t]Q^{*\prime}\left(c_\Sigma^S\right) + q_1^*(\alpha)[1-t] \\ &= -t\alpha t Q^{*\prime}(\xi) + t\alpha 2[1-t]Q^{*\prime}\left(c_\Sigma^S\right) + \left[\frac{P\left(Q^*\left(c_\Sigma^I\right)\right) - \tilde{c}_1}{-P'\left(Q^*\left(c_\Sigma^I\right)\right)}\right][1-t] \qquad \text{(A30)} \\ &> -t\alpha t Q^{*\prime}(\xi) + t\alpha 2[1-t]Q^{*\prime}\left(c_\Sigma^S\right) + \left[\frac{t\alpha}{-P'\left(Q^*\left(c_\Sigma^I\right)\right)}\right][1-t] \qquad \text{(A31)} \\ &\to \frac{-t\alpha t + t\alpha 2[1-t]}{3P'(Q^*(\cdot))} + \frac{t\alpha}{-P'(Q^*(\cdot))}[1-t] \quad \text{as } P''(Q^*) \to 0 \qquad \text{(A32)} \\ &= \frac{\alpha t[2t-1]}{-3P'(Q^*(\cdot))} > 0. \end{aligned}$$

The equality in (A30) and the inequality in (A31) follow from (A28). (A32) follows from (A1). The inequality in (A29) holds because:

$$q_1^{*\prime}(\alpha) \geq Q^{*\prime}(\alpha) = Q^{*\prime}\left(c_\Sigma^I\right)\frac{dc_\Sigma^I}{d\alpha} = -[2-t]Q^{*\prime}\left(c_\Sigma^I\right). \tag{A33}$$

The last equality in (A33) follows from (6). ■

Proof of Corollary 1. Consumer surplus under industry structure $r \in \{S, I\}$ in the presence of downstream Cournot competition is:

$$Z^{rC}(t) = \int_0^{Q^*\left(c_\Sigma^r\right)} \left[P(Q) - P\left(Q^*\left(c_\Sigma^r\right)\right)\right] dQ$$
$$= \int_0^{Q^*\left(c_\Sigma^r\right)} P(Q)\, dQ - Q^*\left(c_\Sigma^r\right) P\left(Q^*\left(c_\Sigma^r\right)\right). \quad \text{(A34)}$$

Substituting from (6) into (A34) provides:

$$Z^{IC}(t) = \int_0^{Q^*\left(2c_h - [2-t]\alpha^I(t) + \bar{c}\right)} \left[P(Q) - P\left(Q^*\left(2c_h - [2-t]\alpha^I(t) + \bar{c}\right)\right)\right] dQ$$
$$> \int_0^{Q^*\left(2c_h - 2[1-t]\alpha^I(t) + \bar{c}\right)} \left[P(Q) - P\left(Q^*\left(2c_h - 2[1-t]\alpha^I(t) + \bar{c}\right)\right)\right] dQ$$
$$\geq \int_0^{Q^*\left(2c_h - 2[1-t]\alpha^S(t) + \bar{c}\right)} \left[P(Q) - P\left(Q^*\left(2c_h - 2[1-t^S]\alpha^S(t) + \bar{c}\right)\right)\right] dQ$$
$$= Z^{SC}(t). \quad \text{(A35)}$$

The strict inequality in (A35) holds because:

$$\{2c_h - 2[1-t]\alpha^I(t) + \bar{c}\} - \{2c_h - [2-t]\alpha^I(t) + \bar{c}\} = t\alpha^I(t) > 0,$$

$Q^{*\prime}(\cdot) < 0$, and $P'(\cdot) < 0$. The weak inequality in (A35) holds because $\alpha^I(t) > \alpha^S(t)$ under Cournot competition when $|P''(Q)|$ is sufficiently small, from Proposition 1. ■

Proof of Proposition 2. Under VS in the presence of Bertrand competition, U's profit given α is:

$$\Pi^{SB}(\alpha) = [w - c^u]Q(w + c_2^S) - K(\alpha) = t\alpha Q(c_h - \alpha[1-t] + c_2^S) - K(\alpha). \quad \text{(A36)}$$

Differentiating (A36) with respect to α provides U's profit-maximizing choice of α, given t:

$$\Pi^{SB\prime}(\alpha) = tQ(c_h - \alpha[1-t] + c_2^S) - t\alpha[1-t]Q'(\cdot^S) - K'(\alpha) = 0. \quad \text{(A37)}$$

Under VI in the presence of Bertrand competition, the integrated firm chooses α to maximize:

$$\Pi^{IB}(\alpha) = [p - (c_h - \alpha + c_1^I)]Q(p) - K(\alpha)$$
$$= [t\alpha + c_2^I - c_1^I]Q(c_h - [1-t]\alpha + c_2^I) - K(\alpha). \quad \text{(A38)}$$

Differentiating (A38) with respect to α provides:

$$\Pi^{IB\prime}(\alpha) = -[1-t][t\alpha + c_2^I - c_1^I]Q'(c_h - [1-t]\alpha + c_2^I) + tQ(\cdot^I) - K'(\alpha)$$

$$= tQ(c_h - [1-t]\alpha + c_2^I) - [t\alpha + c_2^I - c_1^I][1-t]Q'(\cdot^I) - K'(\alpha) = 0. \tag{A39}$$

(A39) implies that $\alpha^{IB} > 0$ for all $t \geq 0$ since $c_2(0) > c_1(0)$ from assumption 1.

(i) When $t \to 0$, (A37) implies that $\alpha^{SB} \to 0$. But from (A39), α^{IB} is bounded strictly above zero because $c_2^I = c_2(0) > c_1(0) = c_1^I$, by assumption 1. Therefore, $\alpha^{IB}(t) > \alpha^{SB}(t)$ when $t \to 0$.

(ii) Define $p^I \equiv c_h - [1-t]\alpha + c_2^I$ and $p^S \equiv c_h - [1-t]\alpha + c_2^S$, so that $p^I - p^S = c_2^I - c_2^S$. The mean value theorem implies that there exists a $\xi \in \left(p^S, p^I\right)$ such that:

$$Q(p^I) - Q\left(p^S\right) = [c_2^I - c_2^S]Q'\left(\xi\right). \tag{A40}$$

In addition, from Lemma 2 and assumption 1:

$$c_2^I \geq c_2^S \geq c_1^S = c_1(L^S) > c_1(0) = c_1^I. \tag{A41}$$

(A41) implies that $c_2^I - c_1^I > c_2^I - c_2^S$. Therefore, from (A37) and (A39):

$$\Pi^{IB\prime}(\alpha) - \Pi^{SB\prime}(\alpha)$$

$$= t\left[Q(p^I) - Q\left(p^S\right)\right] - t\alpha[1-t]\left[Q'(p^I) - Q'(p^S)\right] - [1-t][c_2^I - c_1^I]Q'(p^I)$$

$$= t[c_2^I - c_2^S]Q'\left(\xi\right) - [1-t][c_2^I - c_1^I]Q'(p^I) - t\alpha[1-t]\left[Q'(p^I) - Q'(p^S)\right] \tag{A42}$$

$$> t[c_2^I - c_2^S]Q'\left(\xi\right) - [1-t][c_2^I - c_2^S]Q'(p^I) - t\alpha[1-t]\left[Q'(p^I) - Q'(p^S)\right] \tag{A43}$$

$$\gtrapprox [2t-1]\,[c_2^I - c_2^S]Q'(\xi) > 0, \tag{A44}$$

when $t < 1/2$ and $Q''(p)$ is sufficiently small (so $Q''(p) < 0$ or $Q''(p) \to 0$). The equality in (A42) follows from (A40). The inequality in (A43) follows from (A41), since $Q'(p^I) < 0$. The first inequality in (A44) holds because $0 > Q'(\xi) \gtrapprox Q'(p^I)$ and $0 > Q'(p^S) \gtrapprox Q'(p^I)$ when $Q''(p) \lessapprox 0$. (A44) implies that $\alpha^{IB} > \alpha^{SB}$ for all $t < 1/2$ under the specified conditions because the marginal increase in profit from increasing α is higher under VI than under VS.

(iii) $Q(c_h - [1-t]\alpha + c_2^I) \to Q(c_h - [1-t]\alpha + c_2^S)$ and so $Q'(\cdot^I) \to Q'(\cdot^S)$ when $c_2(0) - c_2(1) \to 0$. Therefore, from (A37) and (A39):

$$\begin{aligned}\Pi^{IB\prime}(\alpha) - \Pi^{SB\prime}(\alpha) &= -t\alpha[1-t]Q'(\cdot^I) + tQ(\cdot^I) - [1-t][c_2^I - c_1^I]Q'(\cdot^I) \\ &\quad - \left[tQ(\cdot^S) - t\alpha(1-t)Q'(\cdot^S)\right] \\ &\to -[1-t][c_2^I - c_1^I]Q'(\cdot) > 0 \quad \text{for } t \in [0,1).\end{aligned} \tag{A45}$$

The inequality in (A45) holds because $c_2^I - c_1^I = c_2(0) - c_1(0) > 0$, from assumption 1. Because the marginal increase in profit from increasing α is higher under VI than under VS, $\alpha^{IB} > \alpha^{SB}$ when $c_2(0) - c_2(1) \to 0$. ■

Proof of Corollary 2. The equilibrium price under vertical industry structure r in the presence of downstream Bertrand competition is (from (1)):

$$w + c_2(L^r) = c_h - [1-t]\alpha^{rB}(t) + c_2^r. \tag{A46}$$

(A46) implies that the corresponding measure of consumer surplus is:

$$Z^{rB}(t) = \int_{c_h - [1-t]\alpha^{rB}(t) + c_2^r}^{\infty} Q(p)\,dp \ . \tag{A47}$$

(A47) implies that for any $t \in [0,1]$:

$$Z^{IB}(t) \gtreqqless Z^{SB}(t) \;\Leftrightarrow\; -[1-t]\alpha^{IB}(t) + c_2^I \lesseqqgtr -[1-t]\alpha^{SB}(t) + c_2^S$$

$$\Leftrightarrow\; c_2^I - c_2^S \lesseqqgtr [1-t]\left[\alpha^{IB}(t) - \alpha^{SB}(t)\right] \;\Leftrightarrow\; \frac{c_2^I - c_2^S}{1-t} \lesseqqgtr \alpha^{IB}(t) - \alpha^{SB}(t) . \tag{A48}$$

From Proposition 2, $\alpha^{IB}(t) > \alpha^{SB}(t)$ when $c_2^I - c_2^S \to 0$. Therefore, (A48) implies that $Z^{IB}(t) > Z^{SB}(t)$ when $c_2^I - c_2^S \to 0$. ■

Proof of Proposition 3. When $t \to 1$, (A37) and (A39) imply:

$$\Pi^{SB\prime}(\alpha) \to Q(c_h + c_2^S) - K'(\alpha) \quad \text{and} \quad \Pi^{IB\prime}(\alpha) \to Q(c_h + c_2^I) - K'(\alpha).$$

$Q\left(c_h + c_2^S\right) > Q(c_h + c_2^I)$ since $c_2^I = c_2(0) > c_2(L^S) = c_2^S$, by assumption 1. Therefore, $\Pi^{SB\prime}(\alpha) > \Pi^{IB\prime}(\alpha)$ when $t \to 1$, and so $\alpha^{SB} > \alpha^{IB}$. ■

Proof of Corollary 3. From Proposition 3, $\alpha^{SB} > \alpha^{IB}$ when $t \to 1$. Therefore, (A48) implies that $Z^{SB}(t) > Z^{IB}(t)$ when $t \to 1$. ■

References

Aghion, Philippe, Nick Bloom, Richard Blundell, Rachel Griffith, and Peter Howitt, "Competition and Innovation: An Inverted-U Relationship," *Quarterly Journal of Economics*, 117(2), May 2005, 701-728.

Arrow, Kenneth, "Economic Welfare and the Allocation of Resources for Invention," in R. Nelson, ed., *The Rate and Direction of Inventive Activity* (Princeton, NJ: Princeton University Press, 1962), pp. 609–625.

Bannerjee, Samiran and Ping Lin, "Downstream R&D, Raising Rivals' Costs, and Input Price Contracts," *International Journal of Industrial Organization*, 21(1), January 2003, 79-96.

Brocas, Isabelle, "Vertical Integration and Incentives to Innovate," *International Journal of Industrial Organization*, 21(4), April 2003, 457-488.

Buehler, Stefan, Dennis Gärtner, and Daniel Halbheer, "Deregulating Network Industries: Dealing with Price-Quality Tradeoffs," *Journal of Regulatory Economics*, 30(1), July 2006, 99-115.

Buehler, Stefan and Armin Schmutzler, "Intimidating Competitors – Endogenous Vertical Integration and Downstream Investment in Successive Oligopoly," University of Zurich Socioeconomic Institute Working Paper No. 0409, July 2005.

Buehler, Stefan, Armin Schmutzler, and Men-Andri Benz, "Infrastructure Quality in Deregulated Industries: Is There an Underinvestment Problem?" *International Journal of Industrial Organization*, 22(2), February 2004, 253-267.

Chen, Yongmin, "On Vertical Mergers and their Competitive Effects," *Rand Journal of Economics*, 32(4), Winter 2001, 667-685.

Chen, Yongmin and Marius Schwartz, "Product Innovation Incentives: Monopoly vs. Competition," University of Colorado working paper, July 2007.

Choi, Jay Pil, Gwanghoon Lee, and Christodoulos Stefanadis, "The Effects of Integration on R&D Incentives in Systems Markets," *Netnomics*, 5(1), May 2003, 21-32.

Choi, Jay Pil and Sang-Seung Yi, "Vertical Foreclosure with the Choice of Input Specifications," *Rand Journal of Economics*, 31(4), Winter 2000, 717-743.

Crandall, Robert and J. Gregory Sidak, "Is Structural Separation of Incumbent Local Exchange Carriers Necessary for Competition?" *Yale Journal on Regulation*, 19(2), Summer 2002, 335-411.

Crew, Michael, Paul Kleindorfer, and John Sumpter, "Bringing Competition to Telecommunications by Divesting the RBOCs," in M. Crew and M. Spiegel eds., *Obtaining the Best from Regulation and Competition.* Norwell, MA: Kluwer Academic Publishers, 2005.

Farrell, Joseph and Michael Katz, "Innovation, Rent Extraction, and Integration in Systems Markets," *Journal of Industrial Economics*, 48(4), December 2000, 413-432.

Gilbert, Richard, "Looking for Mr. Schumpeter: Where Are We in the Competition-Innovation Debate?" in *Innovation Policy and the Economy*, Adam Jaffe, Josh Lerner, and Scott Stern (eds.), Cambridge, MA: National Bureau of Economic Research, 2006.

Gilbert, Richard and Michael Riordan, "Product Improvement and Technological Tying in a Winner-Take-All Market," *Journal of Industrial Economics*, 2007 (*forthcoming*).

Greenstein, Shane and Gary Ramey, "Market Structure, Innovation, and Vertical Product Differentiation," *International Journal of Industrial Organization*, 16(3), May 1998, 285-311.

Gupta, Sudeer and Richard Loulou, "Process Innovation, Product Differentiation, and Channel Structure: Strategic Incentives in a Duopoly," *Marketing Science*, 17(4), Fall 1998, 301-316.

Loury, Glenn, "Market Structure And Innovation," *Quarterly Journal of Economics*, 93(3), August 1979, 395-410.

Reinganum, Jennifer, "A Dynamic Game of R&D: Patent Protection and Competitive Behavior," *Econometrica*, 50(3), May 1982, 671-681.

Salop, Steven and David Scheffman, "Raising Rivals' Costs," *American Economic Review*, 73(2), May 1983, 267-271.

Sandonís, Joel and Ramon Faulí-Oller, "On the Competitive Effects of Vertical Integration by a Research Laboratory," *International Journal of Industrial Organization*, 24(4), July 2006, 715-731.