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PATENT PROTECTION AND STRATEGIC DELAYS IN TECHNOLOGY DEVELOPMENT: IMPLICATIONS FOR ECONOMIC GROWTH

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Abstract

We present an endogenous growth model in which both the investment to develop a new technology—that upgrades the quality of machines—and entry of imitators are determined endogenously. According to the model, how soon the new-technology machine is launched after the patent is granted is influenced by two factors: returns to scale in technology development and "strategic delays." Strategic delays in technology development are most likely to occur when earlier dates of success enable imitators to enter an industry, i.e., when imitation is swift and relatively cheap and/or patent protection is relatively lengthy. We then explore the link between the optimal patent length and economic growth, and find that the equilibrium investment in technology development and thus the expected rate of technological progress exhibit an inverted U-shape relationship with respect to the legal patent length. Our results further suggest that, in order to minimize the strategic delays in technology development and spur technological progress, legal patent lengths should be shorter in industries where barriers to entry are relatively low.

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1. Introduction

Patent protection is an important pillar of the new growth theory. In a static framework, patent holders' ability to extract rents generates textbook inefficiencies due to deadweight loss, but in a dynamic endogenous R&D model, the existence of such rents plays a key role in influencing R&D intensity, new product development, and technological progress.[1] An important extension of the endogenous growth literature highlights how industry market structure affects the patterns of R&D investment and economic growth. As papers in this strand demonstrate, endogenizing the market structure of consumption goods modifies and in some cases altogether alters the empirical implications of the new growth theory.[2]

While the conventional models of new growth theory accentuate the Schumpeterian roles of market size and patent protection in R&D intensity, the structure of markets that could emerge during the length of patent protection is also highly pertinent to the level of monopoly rents associated with R&D investments. In particular, markets would be more competitive in nature so far as earlier dates of success in inaugural product/technology development enable a larger number of imitators to more readily penetrate the industry. This would suggest a natural but fundamental tradeoff: higher product/technology development intensity subsequent to the approval of a patent would lead to not only earlier expected dates of product/technology launch but also the potential loss of monopoly power and watered down returns to technological innovation. Due to this inherent tradeoff, the extent to which R&D intensity is utilized as a means to sustain monopoly power could influence and perhaps dilute the impact of R&D investments on technological progress and economic growth.

In this paper, we develop a model in which both the investment to develop a new technology — that upgrades the quality of machines — and imitators' entry into the competition are determined endogenously. There exists uncertainty with respect to the development date of the new technology, but higher investment shortens the expected development stage.

Once the machines that embed the new technology are introduced, the costs of imitation decline, entry occurs, and the innovator loses its monopoly power. In particular, we consider three phases of imitation driven by its cost: Initially, there is a *dormant* period which covers the length of time prior to the development of an inaugural product

[1]See, for example, Aghion and Howitt (1992, 1996) and Grossman and Helpman (1991a, 1991b).
[2]See, for instance, Aghion et al. (2004) and Peretto (1999a, 1999b).

by its innovator; due to the fact that no product to imitate exists, the cost of imitation is infinite during this initial phase. Secondly, there is an *effective-patent-protection period* when the imitators can only innovate around a patent by observing the qualities and characteristics of the product marketed by the innovator; during this phase, the cost of imitation is large but not infinite.[3] Finally, we get to the *expired-patent* period when imitation becomes costless due to the fact that no patent protection covers the original product. The essential idea here is that, while the patent system is designed to disclose information and make it more easy for potential imitators to invent around a patent, the actual development of (patent-based) products and technologies also helps to reveal additional useful information. There are a number of economically plausible channels through which market entry becomes more feasible over time following the development and marketing of a technology that utilizes a patented idea. These include–but are not confined to–learning-by-observation and reverse engineering.[4] The main concern of our paper is whether there are actions that an innovator can take which deters or subdues costly imitation during the *effective-patent-protection* period. We emphasize the implications of such strategic actions for economic growth.

Using our framework, we are able to reach several novel conclusions. For instance, how soon the new-technology machines are developed and launched after the patent is granted is influenced by three factors. First, as the level of technology improves, the market demand for new machines will rise. Second, if there are decreasing returns to scale in technology development, then the marginal return to technology development

[3]We label this an *effective* protection period because the innovator's idea is protected during the dormant period too; however, since the cost of imitation is prohibitively high when the inaugural product is not yet launched, the protection afforded during the dormant phase is not required in order to deter entry. Put slightly differently, the actual length of the patent covers the dormant phase as well as the effective protection period because, due to competition in the generation of ideas, innovators will apply for patent protection as soon as they have a patentable idea.

[4]Two historical anecdotes can serve to highlight this point. James Watt invented his steam engine in 1765, but until numerous subsequent improvements were made in his design, it remained a crudely engineered piston used principally as a water pump. [See Mokyr (1990, pp. 290-302) for more details.] A more recent example involves the development of the automotive turbo-charger: A Swiss engineer, Alfred J. Beuchi, invented the turbocharger, the basic patent dating from 1905. The application of the turbocharger to high-speed diesels necessitated changes in the unit's design. Manufacturers developed sleeve bearings suitable for extremely high rotating speeds. Using wheel designs suitable for mass production from precision castings, they also developed centripetal, radial in-flow turbines, which had advantages in terms of design simplicity and cost. [See Abernathy and Ronan, 1979.]

For related ideas and their roles in technological change, see Rosenberg (1982) and Mokyr (1990).

will decline as technologies become more sophisticated. If this second returns to scale effect dominates the first market size effect, then length of time between a patent application and the development of an inaugural technology would inevitably widen as technologies mature. In that case, reductions in the length of effective patent protection would be caused by "natural" delays due to diminishing returns to R&D. Third and more interestingly, in deciding how much to spend on new technology development, patent holders would take into account the costs of imitation–and the inherent loss of market power commensurate with those costs. As a consequence, patent holders would adjust their technology development effort in an attempt to maintain their monopoly power over the length of a patent. If indeed patentees reduce their investment in new technology development based on such concerns, then the expected development date of the new-technology machines would again be delayed. Thus, lower technology development intensity based on such concerns would generate "strategic" delays.

Naturally, strategic delays in new technology development are most likely to occur when earlier dates of success in development enable more imitators to penetrate industries. When that is the case, developers would reduce their technology development intensity with the recognition that the sooner is the date of the technology launch, the longer is the amount of time they face competition over the length of the patent. Taking into account both natural and strategic delays, we find that the effective length of patents–the interval of time between the introduction of new technology and the expiration of patents–would be shorter when there exists decreasing returns to scale in technology development and imitation costs are relatively low.

Our model produces some strong normative repercussions as well. In particular, recognizing that there may be strategic delays in new technology development drastically alters how changes in legal patent lengths could influence technology development and economic growth. For one, patent extensions may be growth enhancing if and only if strategic delays are of no major significance. That is, longer patents would unambiguously induce the patentees to raise their technology development effort only when the patentees are confident that the launch of a new technology during an earlier stage of the patent protection period would generate sufficiently generous monopoly profits. This would be more likely when imitation is either costly or it takes a relatively long time. By contrast, extending the length of patents would not be growth enhancing in industries with lower barriers to entry. In such industries, extending the length of patents would generate lower profits for the innovator due to the fact that competition dilutes his or her monopoly rents. And in an attempt to limit the extent of competition via imitation, patentees would resort to strategic delays in technology development by reducing their development effort.[5]

Based on the mechanisms we alluded to above, we find that all three main variables, i.e., the equilibrium amount of technology development investment, the expected level of imitation, and the expected rate of technological progress, follow an inverted U-shape relationship in the length of patents. The reason for this is that longer patents, even though granting the patent holder a potentially longer period of monopoly profits, also entice imitators to enter an industry which in turn induce the patent holder to lower her technology development investment and delay the technology development. As a result of these conflicting effects, equilibrium levels of technology development investment, the expected number of imitators, and the expected rate of technological growth first rise and then fall with longer patents. Indeed, other theoretical work as well as some detailed empirical analyses corroborate such non-monotonic relationships between R&D spending and patent lengths.[6] A final implication of our model suggests that, in order to minimize the strategic delays in technology development and spur technological progress, legal patent lengths should be shorter in industries where barriers to entry are relatively low.

The remainder of the paper is organized as follows: Section 2 reviews the relevant literature. Section 3 describes our model. Section 4 discusses the technology development process and establishes the equilibrium level of technology development investment. Section 5 reviews the equilibrium level of imitation, while Section 6 examines the model's implications for economic growth. And Section 6 concludes.

[5]Implicit in this discussion is the fact that we shall abstract from the scope (or depth) of patent protection. In what follows we solely consider the duration of patent protection as the relevant policy tool, but an alternative to what we develop in this paper could include both the duration and the scope of patents as policy variables. In that case, and when adjusting the length of patents is not a desirable policy option, the latter could be utilized to raise entry barriers in industries in which strategic product development delays are considerable due to swift and easy imitation.

An alternative way to interpret our main findings then is to note that extensions of patent length would not necessarily generate the desired effects on product development efforts unless the former are also accompanied by modifications in patent scope.

[6]See the next section for details.

2. Related Literature

This paper is related to three strands in the existing literature. The first one links endogenous growth models with either endogenous market structure or endogenous entry. In one such example, Aghion et al. (2004) first document that R&D in innovation reacts positively (negatively) to firm entry in technologically advanced (laggard) industries, and then develop a Schumpeterian growth model to backup this empirical finding. Their theoretical results are driven by the fact that, in technologically sophisticated industries, R&D firms can step up their innovative effort to escape entry whereas, in technologically more mature industries, R&D firms lower their effort due to the recognition that they are at a competitive disadvantage vis-a-vis new entrants. The theory we develop below complements the work of Aghion et al. in three different dimensions. One, because we focus on technology development based on existing and active patents–and not on innovations that could potentially generate new patents–we are able to address how imitators' entry into goods production could influence and in turn be determined by deliberate development effort. In contrast, Aghion et al. explore how the threat of entry by one firm could affect the incumbent's innovative effort in technologically sophisticated and laggard industries. Two, the focus on technology development based on active patents (instead of innovations that could generate new patents) also highlights a complementary channel through which economic growth could be affected.[7] That is, the threat of firm entry could not only spur or hamper new innovations (as in Aghion et al.) but it could also influence the speed with which patented ideas are converted to commercial technologies (as in our model). And three, taking the differential impact of firm entry on young and mature industries seriously, we proceed to examine how optimal patent lengths should be tailored to promote economic growth.

The second body of work to which our paper is related focuses on the role of imitation on patenting decisions. The most relevant example in this strand is Gallini (1992) where the imitation of a patented idea is costly and a rival's decision to imitate a patented idea depends on the length of patent protection: the longer the patent, the more likely it is that rivals will invent around a patented idea. The innovator's main decision is to whether to apply for a patent or keep her innovation as a trade secret. Gallini shows that patent extensions need not necessarily generate more patent applications and

[7] As Imbs and Wacziarg (2003) demonstrate convincingly, countries grow rich, at least in part, by expanding the varieties of products and services that they produce.

revelations of new ideas. Due to the fact that longer patents make it more likely that rivals will invent substitute products that lower the option value of a patent, the threat of imitation subsequent to a patent application lowers the incentives to reveal new ideas and raises those to keep them as trade secrets. In Gallini's model, imitation is inevitable once a patent is filed (because filing a patent makes the new idea public knowledge). More importantly, regardless of whether the innovator files for a patent or not, the innovative idea is already embedded in the products the innovator sells on the market (the innovator just weighs the tradeoff between keeping her idea as a trade secret and applying for a patent, revealing the innovative information, but getting patent protection in exchange). Hence, the patenting decision has no effect on the rate of innovation. In our model, this is not the case; we show that there exists an important–but as yet unidentified–interplay between the innovator's decision to develop a technology and the imitators' decision of entry. As a consequence, we are able to investigate the potential impact of costly imitation and market structure on technology development investment and economic growth.

The third strand to which our paper is related examines how R&D expenditures are influenced by patent lengths, time to expiration or patent competition. For example, Kamien and Schwartz (1974) and Goel (1996) show that legal patent lengths and R&D spending are positively and monotonically related. In contrast, Horowitz and Lai (1996) emphasize the theoretical underpinnings of an inverse U-shape relationship between the rates of innovation and legal patent lengths and Lerner (2002) finds empirical support for such a non-monotonic relationship. In a similar vein, Dechenaux, Goldfarb, Shane and Thursby (2003) examine how the commercialization of ideas and the termination of licenses change with the age of patents. Their empirical findings verify that there exists an inverted U-shape relationship between the hazard rates of first commercial sale and the age of patents. Ellison and Ellison (2000) examine whether the behavior of incumbent pharmaceutical companies change in periods close but prior to the expiration of patents. Their empirical findings support the conjecture that strategic intent to deter entry upon the expiration of patents is most pronounced in intermediate size markets (where, unlike small and large markets, both the willingness and ability to deter entry are significant). In another relevant piece, Weeds (2002) shows that the threat of a patent race can generate strategic delays in R&D investment (compared to single firm outcomes), particularly in symmetric and non-cooperative games. What we present below is related to this strand because we examine how the patent length influences technology development intensity. Like Horowitz-Lai and Lerner, we generate an inverted U-shape relationship between patent length and technology development investment. But we differ from those authors because we emphasize the endogeneity of entry through imitation as the source of this inverted U-shape relationship. At the same time, we complement Weeds' findings because we show that potential competition could also generate strategic delays in technology development investment and hinder economic growth.

3. The Economy

3.1 Production

Consider an economy that operates in a world in which a homogenous final consumption good is produced by a continuum of firms indexed by j, $j \in [0, 1]$. The output of each firm j at time t is given by

$$y_t^j = A_t^j \left(l_t^j\right)^\alpha, \quad 0 < \alpha < 1, \tag{1}$$

where A_t^j denotes the endogenously determined "effective" technology level and l_t^j is the number of the workers employed by firm j. By definition, aggregate output, Y_t, equals the sum of output produced by all firms. [In the following section, we explain how the effective technology level, A_t^j, is dependent on the quality and quantity of the machines used in the production, which are in turn determined by the rate of invention.]

The labor market is competitive. Thus, the wage rate paid to labor equals its marginal product:

$$w_t = \alpha A_t l_t^{\alpha-1}, \tag{2}$$

where A_t represents the effective technology level of all firms in aggregate (more on which below).

In equilibrium, the aggregate supply of labor, which is normalized to 1, equals aggregate labor demand (i.e., the total number of workers employed by all firms).

3.2 The Technology

Let q_t^j and m_t^j respectively denote the quality and quantity of machines used in production at time t by a representative firm j. We assume that

$$A_t^j = \frac{\left(q_t^j m_t^j\right)^{1-\alpha}}{1-\alpha}, \tag{3}$$

which implies that potential productivity increases with both the quality and quantity of machines used in production. Because machines depreciate fully in one generation, firms must purchase new machines in every period t.

As in Aghion and Howitt (1992) and in Grossman and Helpman (1991), we assume that when a new discovery is made, the inventor can devote resources to develop machines that embed the new discovery. We assume that this process involves uncertainty with respect to the success date, although higher development investment increases the likelihood of success at creating a new-technology machine at any given time t. When the invention is successfully embedded into the latest generation machines, machine quality (or productivity), q_t improves such that $q_t = (1+g)q_{t-1}$, where $g > 1$.

The decision of the final good production firm j, $j \in [0,1]$, is

$$\max_{m_t^j, l_t^j} A_t^j \left(l_t^j\right)^{\alpha} - p_t m_t^j - w_t l_t^j, \tag{4}$$

where

$$A_t^j = \frac{\left(q_t^j m_t^j\right)^{1-\alpha}}{1-\alpha}, \tag{4a}$$

and p_t denotes the price of a machine of quality q_t. The solution to this problem yields, $\forall j \in [0,1]$,

$$m_t^j = \left[\frac{(q^j)^{1-\alpha}}{p_t}\right]^{1/\alpha} l_t^j. \tag{5}$$

It is important to note that the aggregate demand for machines, which is given by equation (5) aggregated over all j firms, depends strictly positively on their underlying technological sophistication, q_t.

At any given time t, a single firm will own the patent for the machines with the newest technology and sell these machines to consumption goods producers. For older vintages of technology, we assume that any prior patents have expired, and all firms, including both the original patent holder and imitative firms, can produce machines that embed older technology vintages at the constant marginal cost c, $c > 0$.

Given that older vintage of technology are available at a lower price, the monopoly price of machines which embed the newest technologies, p_t, is given by $\min\left[c/\left(1-\alpha\right), gc\right]$, where $c/\left(1-\alpha\right)$ is the unconstrained optimal monopoly markup given the isoelastic demand for machines defined above and gc equalizes the quality-adjusted price of a new machine with that of an older one. For expositional simplicity, we assume hereafter that $g > 1/\left(1-\alpha\right)$ so that consumption goods producers prefer to buy the newest technology even when the monopolist charges the unconstrained price $c/\left(1-\alpha\right)$.[8]

3.3 Technology Development

When a new discovery is made, the inventor is granted a patent to develop its applications. The patent lasts $\mathcal{L}$, $\mathcal{L} > 0$, periods. Each new idea serves as the basis of one new technology that can potentially improve the quality of machines. The inventor of a new idea can choose to invest resources to develop machines which embed the new idea.[9] There is uncertainty with respect to the timing of the successful development of machines with the new idea, but higher development spending shortens the expected technology development stage. Once the new technology is introduced, the costs of imitation decline and competitors can, with some time lag, develop substitutes for the newest technology.

Let d_t represent the level of development effort carried out by the inventor to develop a new-technology machine. The only input of the development effort is the final consumption good. Also let $t_0(d_t, q_t)$ denote the stochastic success date for developing a new-technology machine, i.e., q_{t+1}, when the current quality level of the machines equals

[8] Our qualitative conclusions would not be altered if innovations were more incremental in nature. The only difference then would be reflected in a lower price for inaugural products in the market: instead of $c/(1-\alpha)$ newest-technology goods would sell at a price of gc, where $gc < c/(1-\alpha)$.

[9] It is also possible that a patent holder licenses its idea to be used by others to develop a new technology based on his or her patent. The main conclusions we discuss below would not be altered as long as licensees bid for the patent-holder's idea in a perfectly competitive fashion. In that case, all the rents associated with producing the technology under the patent protection would still accrue to the patent holder.

q_t. Then, the probability of developing a new-technology machine at or before time t is, $\forall\, t \in [0,\, \infty)$,

$$\Pr[t_0(d_t, q_t) \leq t] = 1 - \exp[-h(d_t, q_t)t]. \tag{6}$$

Based on the above equation, the expected success date for the technology developer is $1/h(d_t,\, q_t)$ where $h(d_t,\, q_t)$ is the hazard function in new-technology machine development. By assumption, the hazard rate function $h(d_t,\, q_t)$ is such that the probability density function of new-technology machine development is strictly concave in d_t with $h_1 > 0$, $h_2 \leq 0$, $h_{11} \leq 0$, $h_{12} \leq 0$, and $h_1^2 + hh_{11} > 0$.[10]

In order to derive the expected profits of a technology development firm, we first need to address the imitators' endogenous entry decision once the patent holder succeeds in developing the inaugural new-technology machine. At first, the cost of developing the inaugural new-technology machine is prohibitively large for all those who do not own the patent. But after τ, $\tau > 0$, periods following the development of the first application based on the patent, the imitation cost declines to F, $F > 0$. This assumption is motivated by the idea that imitation becomes feasible with a lag after a new-technology machine becomes commercially available. Recalling that t_0 denotes the period in which the new-technology machine hits the market, the total profits of the innovator drops to F in present value when and if imitation begins, which could occur in period t_1, $t_1 \equiv t_0 + \tau$.

The imitators' entry decision, which influences whether the patent holder is able to sustain its monopoly power, is implicit in the imitation cost F and its lag τ. Given that the date of a new technology development, t_0, is stochastic, the actual number of imitators that would emerge in a given industry, z_t, are also stochastic ex ante. But, once t_0 is realized, entry in period t_1 will exhaust the present value of discounted profits. As a result, the following equilibrium condition will obtain:

[10]The strict concavity of the p.d.f. $f(d_t, q_t, t) \equiv h(d_t, q_t)\ \exp[-h(d_t, q_t)\ t]$ is ensured when $h_{11} < 0$ and $h_1^2 + hh_{11} > 0$. Then, as we show in Appendix section A.1, the equilibrium amount of technology development investment is unique. The property $h_1 > 0$ captures the fact that the inter-temporal probability of a successful invention is increasing in technology development effort d_t; $h_2 \leq 0$ covers both the case in which the inter-temporal probability of new technology development effort is independent of the state of the technology (i.e. $h_2 = 0$) as well as the case in which it is decreasing in the state of the technology (i.e. $h_2 < 0$).

$$\frac{V_{t_0}[\exp(-\rho t_1) - \exp(-\rho \mathcal{L})]}{\rho[1 + z(t_0)]} = F \tag{7}$$

where the left hand side of the equation represents the present value of the patentee firm's cumulative profit from t_1 to $\mathcal{L}$. The technology developer, who owns the patent for the new-technology machines, makes a profit by selling its machines to the final good producers. A monopolist who introduces his new-technology machines to the market at date t_0 makes a total profit of $V_{t_0} \equiv \sum_j (p_{t_0} - c) m_{t_0}$. Taking into account equation (5), the monopoly pricing equation $p_{t_0} = c/(1-\alpha)$, and the labor market clearing condition $\sum_j l_j = 1$, we find

$$V_{t_0} = \alpha \left[\frac{(1-\alpha)q_{t_0}}{c}\right]^{(1-\alpha)/\alpha}. \tag{8}$$

Then solving equation (7), we get the number of imitators that will emerge given the success data of technology development t_0:

$$z(t_0) = \max\left[0, \frac{V}{F\rho}\left[\exp(-\rho(t_0 + \tau)) - \exp(-\rho\mathcal{L})\right] - 1\right]. \tag{9}$$

Equation (9) implies that there exists a date of discovery, $t_0 \leq \mathcal{L} - \tau$, after which no imitator would choose to enter the industry. That is, if the inaugural new-technology machine is introduced late enough, no imitator would find it profitable to enter with a lag of τ periods given the imitation cost of F. Letting T denote this date, setting the term on the right hand side of (9) equal to zero, and solving it for T yields

$$T = -\frac{1}{\rho}\log\left[\exp(-\rho\mathcal{L}) + \frac{F\rho}{V}\right] - \tau. \tag{10}$$

According to (10), the threshold date of launching the new-technology machines (that deters all imitators) asymptotically approaches $\mathcal{L} - \tau$ as the fixed cost of imitation F approaches zero. In other words, in the absence of an imitation cost, when the inaugural new-technology machine is launched before $\mathcal{L} - \tau$, imitators will emerge in a lag of τ periods. But, if an inaugural new technology is introduced at or after date $\mathcal{L} - \tau$, there will be no imitators throughout the remaining patent period $\mathcal{L} - t_0$. In essence, the effective patent protection only lasts τ periods at the maximum. By contrast, when the fixed imitation cost F is positive, this threshold date of launching the new-technology machines (that deters all imitators) is earlier than $\mathcal{L} - \tau$, and the maximum period of effective patent protection exceeds τ periods. In fact, the higher the fixed cost of imitation, the earlier the threshold technology launch date T after which there will be no imitation and the longer is the effective patent protection period the patentee enjoys.

A patentee firm proceeds by choosing the amount of technology development investment it plans to undertake in each period t, d_t. At any t, such a firm solves the following maximization problem:

$$\max_{d_t} \Pi(t) = \int_t^T \pi_{t_0}^C h(d_t, q_t) \exp[-h(d_t, q_t)t_0] dt_0 + \int_T^{\mathcal{L}} \pi_{t_0}^N h(d_t, q_t) \exp[-h(d_t, q_t)t_0] dt_0 \ , \tag{11}$$

where the two terms on the right hand side respectively denote the expected present discounted value of introducing the inaugural new-technology machines before T (i.e., $t_0 < T$) and that of introducing them after T but before $\mathcal{L}$ (i.e., $T \leq t_0 < \mathcal{L}$). In particular, when the patent holder launches the new-technology machines before T, the present discounted value of her profit stream is

$$\pi_{t_0}^C = V_{t_0} \int_{t_0}^{t_1} \exp(-\rho s) ds \ + \ F \ - \ f \int_t^{t_0} d_t \exp(-\rho s) ds; \quad t_0 \ < \ T \ , \tag{12}$$

where f $(f > 0)$ is the per-period opportunity cost for each unit of the resources tied up in technology development research. Because $t_0 < T$, imitators would enter the market prior to the expiration of the patent protection (from the period $t_1 \equiv t_0 + \tau$), and as a result the present discounted value of the profits the patent holder would receive in the remaining periods is diluted to the level of the imitation cost F, as established in equation (7). When $T \leq t_0 < \mathcal{L}$, the imitators would find it unprofitable to enter prior to the expiration of the patent. As a consequence, the patent holder would be able to sustain her monopoly power until the patent expires. Thus, the present discounted value of profits the patent holder would receive is

$$\pi_{t_0}^N = V_{t_0} \int_{t_0}^{\mathcal{L}} \exp(-\rho s) ds \ - \ f \int_{t}^{t_0} d_t \exp(-\rho s) ds; \quad T \leq t_0 < \mathcal{L}. \tag{13}$$

Clearly, the expected profits of the patentee firm, given by equations (11) through (13), could be influenced by many factors, but the barriers to entry and the length of patent protection are of utmost importance.

4. Equilibrium Development Effort

The problem of the patent holder is to maximize equation (11) with respect to the technology development effort d_t, taking as given equations (8), (10), (12), (13) and the quality level q_t. The first-order condition for this problem satisfies

$$\left. \begin{array}{r} \int_t^T \pi_{t_0}^C \left[1 - h(d_t, q_t)t_0\right] h_1 \exp[-h(d_t, q_t)t_0] dt_0 \\ + \int_T^{\mathcal{L}} \pi_{t_0}^N \left[1 - h(d_t, q_t)t_0\right] h_1 \exp[-h(d_t, q_t)t_0] dt_0 \\ + \int_t^T h(d_t, q_t) \exp[-h(d_t, q_t)t_0] \frac{\partial \pi_{t_0}^C}{\partial d_t} dt_0 \\ + \int_T^{\mathcal{L}} h(d_t, q_t) \exp[-h(d_t, q_t)t_0] \frac{\partial \pi_{t_0}^N}{\partial d_t} dt_0 \end{array} \right\} \leq 0 \tag{14}$$

where

$$\frac{\partial \pi_{t_0}^C}{\partial d_t} = \frac{\partial \pi_{t_0}^N}{\partial d_t} = -f \int_t^{t_0} \exp(-\rho s) ds < 0 \ . \tag{15}$$

The first two terms in equation (14) define the benefit of a marginal increase in technology development effort and the last two terms define its cost. The marginal benefit of technology development effort represents the increase in expected profits due to an increase in d_t. According to those two terms in (14), an increase in d_t helps to expedite the launch of the new-technology machines and prolong the effective patent length, $\mathcal{L} - t_0$, which may potentially extend the time period over which the patent holder can exercise some degree of market power. The marginal cost of technology development is given by the last two terms in (14) and equation (15). The optimal level of technology development investment given by (14) and (15) is, of course, dependent on the current time period t and time left to expiration, $\mathcal{L} - t$. The next proposition details and summarizes related observations.

Proposition 1 *The profit-maximizing technology development investment, d_t^*, attains an interior maximum and an inverted U-shape pattern with respect to t.*

Proof: See Appendix Section A.1.

Proposition 1 indicates that, for a given level of imitation cost F, the incentive to develop a new-technology machine first rises and then falls as the expiration date approaches. When there is a sufficiently long time to expiration, new machine developers have no incentive to rush since launching the new technology at an earlier date would entice imitation. As the patent expiration date nears, the threat of imitation declines but so does the monopoly rent associated with the new machine launch. These two conflicting forces generate a peak in technology development investment sometime during the patent protection period, where the peak is associated with an expected technology launch date that delays imitative entry and yields relatively more monopoly power.

It is important to note that the marginal benefit of increasing the technology development effort depends on the ease of imitation in and entry into the technology market. As equations (11)-(13) show, the innovator either enjoys being the sole monopolist of this technology for a longer period of time or shares the monopoly rents with only a very restricted number of firms when imitation is rather difficult (i.e., when either τ is large, F is large or both). By contrast, when imitation is relatively easier (when both τ and F are relatively small), the innovator's monopoly rents get diluted sooner and more severely. As it will become apparent below, most of our main results are related to this observation.

Proposition 2 *The profit-maximizing technology development investment, d_t^*, is strictly increasing in*

(i) *the imitation cost F;*

(ii) *the imitation lag τ.*

Proof: See Appendix Section A.2.

Figure 1 illustrates our finding that the equilibrium intensity of technology development would first rise and then fall with extensions in patent lengths, and that a rise in the imitation cost, F, shifts the curve upward raising the intensity of technology development at any given patent length. In an extreme case, when the imitation cost is prohibitively high or the patent length is inadequate to compensate the imitation cost (denoted as $\mathcal{L} < \mathcal{L}_0$), there will be no imitation and hence no strategic delays, as further demonstrated in the next section.

[Figure 1 about here.]

Returns to scale in technology development, market size and competition that could eventually emerge all play important roles in determining the intensity of technology development. Together, they influence the extent to which technology development is delayed and the length of effective patent protection changes over the long run. In particular, there are two potential sources of delay in technology development according to this framework. The first one arises when the probability of success in technology development is decreasing in the underlying level of technology ($h_2 < 0$) and this dominates the larger market size effect of better technologies, there is a "natural delay" over time in new technology development. The second potential source of delay in technology development is attributed to the threat of imitation which can manifest itself in diluted monopoly rents for the patent holder. That is, the ease with which imitators can enter the market is important because developers take into account how their expected timing of success influences the competition they face in the future and whether their monopoly power will be weakened.

When $h_2 = 0$ so that the marginal success rate of developing new technologies depends only on the intensity of development effort and not on the state of the underlying level of technology, the "natural delay" in technology development is absent while the patent holder's "strategic delay" incentive is crucial in determining the optimal technology development effort. Not surprisingly, the optimal technology development effort will be higher in industries where either the imitation cost or its time lag is large enough to restrict the potential number of entrants. Put differently, as long as the imitation cost is prohibitively high or it takes a relatively long period of time to imitate, a patent holder will have all the incentives to develop the technology as soon as possible because her monopoly profits would not be diluted once she is successful. In that event, delays in technology development effort due to strategic reasons play a minimal to negligible role.

When $h_2 < 0$ so that the marginal success rate of developing new technology also depends negatively on the state of the technology, the equilibrium amount of effort, d_t, will adjust with changes in the underlying technology level. As it becomes more difficult to develop new technologies, the expected return to technology development effort will decline with improvements in the level of technology if the negative effect of quality upgrading on marginal rate of success is not completely offset by its positive effect on the patentee firm's ex-post profit. Moreover, the delay in the introduction of inaugural technologies and the rate at which the effective length of patents, $\mathcal{L} - t_0$, decreases will depend on the ease of imitation in that industry. That is, the combination of natural delay as a result of decreasing marginal rate of success in technology development and strategic delay due to expected imitation will influence the degree to which the effective patent lengths narrow.

Patent extensions could mitigate the adverse effects of natural delay by allowing new machine developers who own the patents to recover their investments over longer protection periods, but they may exacerbate the patentee firm's strategic delays. Holding constant the development date of a machine with the latest technology, longer patents would attract imitators into the market and dilute the monopoly rents of the patent holder. That, of course, would be the case when entry barriers are low (i.e. when F and τ are both relatively low). In such cases, patentees would respond by reducing their technology development effort and by strategically delaying technology development. By contrast, patent extensions may induce patentees to raise their technology development effort only when the discovery of a new technology during an earlier stage of the patent protection period generates a longer interval of time during which the innovator enjoys either all the monopolistic rents or–if there is imitation early on–close to all of the monopolistic rents. This would occur in industries with high entry barriers (i.e., where F or τ is high). Proposition 3 summarizes this finding.

Proposition 3 *For the profit maximizing patentee firm,*

(i) d_t^* *is strictly increasing in the legal patent length, $\mathcal{L}$, when the imitation cost is relatively high ($\forall\ F > \underline{F}$) and strictly decreasing in the legal patent length, $\mathcal{L}$, when the imitation cost is relatively low ($\forall\ F < \underline{F}$);*

(ii) d_t^* *has an inverted U-shape with respect to the legal patent length, $\mathcal{L}$, and reaches maximum at $\mathcal{L}_1$;*

(iii) *the skewness of this curve rises with the imitation lag, τ.*

Proof: See Appendix Section A.3.

In Figure 2, we show how technology development spending is influenced by market power considerations and how changes in legal patent length $\mathcal{L}$ could affect optimal investment in new machine development. In the two panels of Figure 2, we plot the relationship between the current time period t and the expected duration of monopoly (denoted by $\mathcal{L}_m$) relative to the length of the remaining patent period ($\mathcal{L} - t_0$), i.e., $E(\mathcal{L}_m/\mathcal{L} - t_0)$. In other words, we consider the fraction of the remaining patent period over which the innovator enjoys its monopoly status as a measure of monopoly power. In this measure, the duration of monopoly, $\mathcal{L}_m$, either equals $t_1 - t_0 = \tau$ if $t_0 < T$ or $\mathcal{L} - t_0$ if $t_0 \geq T$. As seen in Figure 2.a, if the technology is developed at or after the threshold date T, all imitators will be deterred from entry and the original technology developer will enjoy monopoly throughout the remainder of the patent protection period. Before we reach the threshold date T, the further we get into the monopoly protection period and the inaugural technology is not yet developed, the greater is the fraction of the remaining patent period over which the original technology developer will enjoy monopoly. The dashed curve in Figure 2.a depicts the impact of lower technology development spending, d_t, on the expected duration of monopoly during the patent protection period. As shown, a reduction in technology development spending delays the development of technology, but it also extends the duration of monopoly relative to the remaining protection period. In Figure 2.b, we show how changes in patent length, through their impact on the innovator's optimal technology development investment, influence the expected duration of monopoly relative to the length of the remaining patent period, $E(\mathcal{L}_m/\mathcal{L} - t_0)$. The solid curve shows the benchmark case in which we hold constant the initial technology development investment. The dashed curve incorporates the adjustments in the optimal level of technology development investment: as we demonstrate in Proposition 3, it reflects an increase in d_t^* with higher $\mathcal{L}$ when $\mathcal{L} < \mathcal{L}_1$ and a decrease in d_t^* with higher $\mathcal{L}$ when $\mathcal{L} \geq \mathcal{L}_1$. In particular, only when $\mathcal{L} \geq \mathcal{L}_1$, the dashed line lies above the solid line, reflecting the fact that strategic delays kick in at longer patent lengths.

[Figures 2.a and 2.b about here.]

5. Equilibrium Imitation Level

We now examine if the patent holder's optimal development strategy would successfully block imitators so that $z(t_0) = 0$. Recall that, since the date of a new technology development, t_0, is stochastic, the actual number of imitators that would emerge in a given industry, z_t, is also stochastic. Still, in an ex ante sense, more can be said about the anticipated level of imitation and the factors that influence it.

In expected terms, the potential number of entrants given by equation (9) can be defined as

$$E_t[z(t_0)] = \int_t^T z(t_0)h(d_t, q_t) \exp[-h(d_t, q_t)t_0]dt_0. \tag{16}$$

The expected equilibrium number of imitators, $E_t[z(t_0)]$, rises with technology development investment, d_t. The underlying reason for this is fairly evident: a higher development effort by the innovator would lead to an earlier expected date of success in developing the new technology, thereby raising the probability for imitators to free ride this success over a longer period of time.

Lemma 1: *The number of expected imitators, $E_t[z(t_0)]$, is strictly increasing in the innovator's technology development investment, d_t^*.*

Proof: See Appendix Section A.4.

Such free-riding incentives would be difficult to realize in industries with relatively high barriers to entry. In fact, the impact of entry barriers, F, on the equilibrium number of expected imitators, $E_t[z(t_0)]$, consists of two factors, one that is *direct* and the other that is *indirect.* Entry barriers discourage imitation directly by making imitation more costly. At the same time, however, they bring closer the expected date of inaugural technology development. This is due to the fact that higher barriers to entry prolong the innovator's effective patent protection and entice her to invest more in technology development. This in turn encourages more free-riding by the imitators. The overall impact of changes in F on imitation, therefore, depends on which of the above two factors dominates.

The legal patent length also influences the equilibrium level of imitation. On the one hand, a longer patent length would allow imitators to enjoy some degree of imperfect competition for an extended period of time thereby engendering more imitation. This is the *direct* effect of longer patents on imitation. On the other hand, a longer patent generates an *indirect* effect through the equilibrium level of technology development investment. This is due to the fact that longer patents encourage more technology development investment when the entry barriers are sufficiently high, in which case they also bring forward the expected date of the technology development and induce more (but still relatively few) imitators to free ride such effort. Then, both the direct and indirect forces affect the emergence of imitation in the same direction and we conclude that patent extensions would encourage imitation despite the fact that the barriers to entry are relatively high (i.e., when $F \geqslant \underline{F}$). However, when the barriers to entry are significantly low (i.e., when $F << \underline{F}$), extending patent lengths would only discourage the equilibrium technology development effort and reduce the probability of a free ride. In general, if the indirect effect dominates, the number of imitators would decrease when the legal patent length is extended.

Proposition 4 *The expected number of imitators,* $E_t[z(t_0)]$*,*

(i) *rises with a longer legal patent length when the imitation cost exceeds a certain level,* $\underline{F}$*; and it falls with a longer legal patent length when the imitation cost is sufficiently lower than* $\underline{F}$*.*

(ii) *has an inverted U-shape with respect to the legal patent length, $\mathcal{L}$, when there exists some imitation, and it reaches a maximum at $\mathcal{L}_2$ (with $\mathcal{L}_2 < \mathcal{L}_1$).*

Proof: See Appendix Section A.5.

An implication of this proposition is that, for any given level of imitation cost F and imitation lag τ, the expected number of imitators would first rise and then fall with extensions in patent lengths. Figure 3 depicts this result. As shown, when the patent length is very short and inadequate to compensate imitators' fixed cost of imitation (i.e., $\mathcal{L} < \mathcal{L}_0$), imitators would choose not to enter. However, when the patent length is sufficiently long (i.e., $\mathcal{L} \geq \mathcal{L}_0$), imitation is expected to emerge. Furthermore, as the patent length is extended, the expected number of imitators starts to rise until the point $\mathcal{L} = \mathcal{L}_2$ where the innovator has substantially reduced its technology development effort. A higher imitation cost, F, shifts downward the curve relating $E[z(t_0)]$ to $\mathcal{L}$ and reduces the number of imitators for any given level of legal patent length $\mathcal{L}$.

[Figure 3 about here.]

6. Patent Protection and Growth

Finally, we can elaborate on how patent protection also affects the pace of economic growth through its impact on the patent holder's technology development effort and the "strategic delay" incentive. Economic growth manifests itself primarily in two variables in our economy: the quality of machines, q_t, and the rate of technological progress, g_t, where $q_{t+1} = (1 + g_t)q_t$ and

$$g_t = \begin{cases} g & \text{with hazard rate function } h(d_t, q_t), \\ 0 & \text{otherwise.} \end{cases} \tag{17}$$

Since the hazard rate of technology development success decreases with the underlying technological quality level, extending patent protection may lead to technological progress and thereafter enhance economic growth only if it induces the patentee to devote a greater amount of technology development effort. Again, this could only happen when the incentive of strategic delays is of no major significance. That is, when the patent holders are sufficiently protected from imitators by high entry barriers. By contrast, in industries with low entry barriers, holding constant the development date of an inaugural technology, longer patents would only raise the likelihood of competition from imitators and/or prolong the time period during which the innovator has to share its profit with competitors. Hence, with low entry barriers (i.e. where F and τ are low), patentees would respond by reducing their technology development effort, which in turn lowers the probability of technological progress and pace of economic growth.

We can hold constant the level of imitation costs and examine how the legal patent length affects economic growth. Taking into account the probability density function of the technology development, i.e., $h(d_t, q_t)\exp[-h(d_t, q_t)t]$ derived from equation (6) and equation (17), we obtain

$$E(g_t) = g \times h(d_t, q_t)\exp[-h(d_t, q_t)t], \tag{18}$$

where $h(d_t, q_t)\exp[-h(d_t, q_t)t]$ is an increasing and concave function of d_t given that $h_{11} \leq 0$ and $h_1^2 + h_{11} > 0$. Combining the above property of the probability density function and part (ii) of Proposition 3, we find that the expected pace of quality improvements and growth, $E(g_t)$, similarly exhibits an inverted U-shape relationship with respect to the legal patent length. We depict this relationship in Figure 4. The optimal patent length that maximizes the patentee's technology development investment also maximizes the rate of technological growth. However, when the patent length is too short or too long, the patentee may choose to devote little or even zero effort to developing the inaugural technology, which leads to sluggish or no technological progress.

[Figure 4 about here.]

In Figure 5, we summarize the policy implications of our main conclusions for growth. When the imitation cost is considerably large and the patent length is short and inadequate to compensate for the fixed imitation cost (i.e., $\mathcal{L} < \mathcal{L}_0$), there is no imitation and hence no strategic delays as suggested in region I of Figure 5.[11] When

[11]Note that $\mathcal{L}_0$ represents the threshold patent length for which the expected number of imitators

the patent length is a little longer (i.e., $\mathcal{L}_0 \leq \mathcal{L} < \mathcal{L}_1$) but the imitation cost is still relatively large as shown in region II of Figure 5, there will be no strategic delays in technology development investment, even though there exists a certain degree of imitation which emerges towards the end of patent protection.[12] In regions I and II, patent extensions would stimulate more technology development investment and technological progress because of sufficiently high entry barriers. When imitation is relatively easy and costless and the patent length is rather long (i.e., $\mathcal{L} \geq \mathcal{L}_1$), there will be imitation and, in response, strategic delays by the innovators in technology development investment, as shown in region III of Figure 5. Extending the length of patents in such cases would lower technology development investment and delay technological progress. In sum, for any given level of imitation cost F, the most growth-enhancing level of patent length $\mathcal{L}$ would be determined by the upper bound of region II in Figure 5. It is noteworthy to point out that, while we have solely considered the length of patent protection as the relevant policy tool, we could interpret the level of imitation cost as an alternative tool of patent policy, the patent scope. To a great extent, expanding patent scope would raise the imitators' entry barriers. In this context, the upper bound of region II in Figure 5, (i.e., $\mathcal{L}_1 = g_1(F)$ which we derive in Appendix A.3), represents the optimal combination of the two aspects of patent policy–length and scope–which maximizes the rate of the technological growth.

[Figure 5 about here.]

We conclude this discussion by noting that costly imitation is not necessarily a socially wasteful activity. On the one hand, it might delay the introduction of new-technology products in the market and act as an impediment to economic growth. But, on the other hand, it generates price competition among the producers of existing products and helps to improve consumer welfare. Hence, whether or not costly imitation and the existence of strategic product development delays in response to it can be welfare reducing will depend on model fundamentals, such as consumer preferences and cost of imitation.

is zero, and is an increasing function of F as shown in Figure 3. In Figure 5, we denote this locus as $\mathcal{L}_0 = g_0(F)$. The intuition is that, with higher entry barriers, the threshold patent length required to break even is longer.

[12]Recall that $\mathcal{L}_1$ represents the optimal patent length that maximizes the innovator's technology development investment. As shown in Figure 1, this is also an increasing function of F. In Figure 5, we denote this locus as $\mathcal{L}_1 = g_1(F)$.

7. Conclusion

The novelty of the theoretical model we developed above is that both the investment to develop new technology and imitators' entry into the competition are determined endogenously. There exists uncertainty with respect to the development date of an inaugural technology, although greater investment shortens the expected technology development stage. Once an inaugural technology is introduced, the costs of imitation decline.

On the basis of this model, we are able to reach several important conclusions. First, how soon the inaugural technology is launched after the patent is granted is influenced by two factors. On the one hand, if the marginal return to new technology development is decreasing in the state of the existing technologies and it exceeds the positive market size effects of improvements in technology, then reductions in the length of effective patent protection (which we define as the time between a patent application and the introduction of inaugural machines based on the patent) are caused by "natural" delays. On the other hand, in deciding how much to spend on new technology development, patent holders take into account how imitation costs influence the degree of technology market competition once an inaugural technology hits the market. If imitation is relatively easy, then there are "strategic" delays in new technology launches as well. Taking into account these two factors, the effective length of patents would be shorter when there exists decreasing returns to scale in technology development and imitation costs are relatively low.

Second, when there are potential strategic delays in new technology development, changes in the length of patent protection could influence technology development intensity and the pace of economic growth in different ways. In particular, when the marginal return to technology development is decreasing in the state of the existing technologies, patent extensions may be growth enhancing if and only if strategic delays are not prevalent. By contrast, extending the length of patents would not be growth enhancing when entry barriers are low. Then, holding constant the development date of an inaugural technology, lengthier patents, while supposedly extending the legal protection of monopoly rents over a longer period, would also attract entry by imitators. And in an attempt to keep stable the period over which they enjoy a pure monopoly status, patentees would respond by reducing their technology development effort and, hence, by strategically delaying the anticipated development date of their technology. At a balance of these two conflicting effects, the equilibrium intensity of technology development and hence the expected rate of technological progress exhibit an inverted U-shape relationship with respect to the length of legal patent.

8. Appendix

A.1 Proposition 1: The profit-maximizing technology development investment, d_t^*, attains an interior maximum and has an inverted U-shape with respect to t.

Proof: When $d_t = 0$ (and hence $h(d_t, q_t) = 0$), the first-order condition (denoted as FOC) can be simplified as

$$\left\{ \begin{array}{c} \{V[-\exp(\mathcal{L}\rho) + \exp(\rho(\mathcal{L}+\tau)) - \exp(\rho(t+\tau))\rho(\mathcal{L}-T)] \\ +\ F\rho\gamma[1+\rho(T-t)]\}h_1(0,q_t) \end{array} \right\} \Big/ \ \gamma\rho^2 \quad > \quad 0, \tag{a.1}$$

where $\gamma \equiv \exp[\rho(\mathcal{L}+t+\tau)]$, $h_1(0,q_t) > 0$, and $|\rho\ (T - \ t\ )| < 1$ due to the fact that $\rho(\mathcal{L}-T) < 1$ and $t < \mathcal{L}$.

When $h_1 \to 0$, the first-order condition is

$$\frac{\exp[-(\mathcal{L}+t)(\rho+h)]\{-\exp[\mathcal{L}(\rho+h)]\rho + \exp(th)[-\exp(t\rho)h + \exp(\mathcal{L}\rho)(\rho+h)]\}}{\rho(\rho+h)}. \tag{a.2}$$

The first-order condition specified in (a.2) is negative only if $-\exp[\mathcal{L}(\rho+h)]\rho + \exp(th)[-\exp(t\rho)h + \exp(\mathcal{L}\rho)(\rho+h)] < 0$, the derivative of which with respect to t is equal to $h(h+\rho)[\exp(\mathcal{L}\rho+th) - \exp(t\rho+th)] > 0$ because $t < \mathcal{L}$. When $t = \mathcal{L}$, the first-order condition in (a.2) is 0. Thus, $(a.2)$ is negative $\forall t < \mathcal{L}$.

Next we note that the profit function $\Pi(t)$ is continuous and strictly concave in d_t and that the support of d_t is closed and bounded from below and above at 0 and V/ρ respectively. Together with equations (a.1) and (a.2), we find that the function Π_t attains a unique and interior maximum. As a consequence, we establish that $\partial FOC/\partial d_t$, evaluated at the optimal level of d_t, is strictly negative.

According to the implicit function theorem, the sign of $\partial d_t/\partial t$ depends on $\partial FOC/\partial d_t$ and $\partial FOC/\partial t$, where

$$\frac{\partial FOC}{\partial t} = \frac{\exp[-\rho(t+\tau)-h(\mathcal{L}+t)]}{\rho}\left\{\begin{array}{l}\exp(\rho\tau)[\exp(\mathcal{L}h)-\exp(th)]\rho \\ + \\ [\exp(\mathcal{L}h)V-\exp(\rho(t+\tau)+\mathcal{L}h)F\rho \\ + \\ \exp(\rho\tau+th)\mathcal{L}d_t\rho-\exp(\rho\tau+\mathcal{L}h)(V+d_t t\rho) \\ + \\ \exp(\mathcal{L}h)t[(\exp(\rho\tau)-1)V+\exp(\rho(t+\tau))F\rho]h]h_1\end{array}\right\} \quad \text{(a.3)}$$

When $t = 0$, $\partial FOC/\partial t = 1 - \exp(-\mathcal{L}h) + [\exp(-\mathcal{L}h)\mathcal{L}d_t - F - \tau V]h_1 > 0$ because $F + \tau V < \exp(-\mathcal{L}h)\mathcal{L}d_t$ which ensures the break-even d_t is reached at $t < \mathcal{L}$. When $t = \mathcal{L}$, τ and F are no longer relevant and, since $1/h[d_t^*(\mathcal{L}), q_t] < \mathcal{L}$, we have $\partial FOC/\partial t = [h_1(\mathcal{L}h - 1)V] \ / \ \rho\delta < 0$, where $\delta \equiv \exp[\mathcal{L}(\rho + h)]$. Next we note that the FOC is continuous in t and that the support of t is closed and bounded from below and above at 0 and $\mathcal{L}$ respectively. Thus, the function FOC attains an interior maximum where $\partial FOC/\partial t = 0$. As a result, $\partial^2 FOC/\partial t^2$, evaluated at the optimal level of t, is strictly negative. Recall $\partial d_t^*/\partial t = -[\partial FOC/\partial t]/SOC$, where SOC ($\equiv \partial FOC/\partial d_t$, the second-order condition) at d_t^* is strictly negative as established in (i). Hence, the sign of $\partial d_t^*/\partial t$ is consistent with the sign of $\partial FOC/\partial t$. Since $\partial FOC/\partial t$ is first positive and then becomes negative as t rises, d_t^* also attains an interior maximum with respect to t and exhibits an inverted U-shape. ■

A.2 Proposition 2: The profit-maximizing technology development investment, d_t^*, is strictly increasing in

(i) the imitation cost, F;

(ii) the imitation lag, τ.

Proof: (i) According to the implicit function theorem, the sign of $\partial d_t/\partial F$ depends on $\partial FOC/\partial F$ where FOC denotes the first-order condition:

$$\frac{\partial FOC}{\partial F} = h_1 \exp(-th)\{\exp[h(t-T)]T - t\}, \quad \text{(a.4)}$$

where $h_1 > 0$. Taking the derivative of $\exp[h(t-T)]T - t$ with respect to t, we get $Th\exp[(t-T)h] - 1 < 0$ for $Th\exp[(t-T)h] < \exp[(t-T)h] < 1$ given that $t < T$. Because the rest of the argument also falls with a larger t, the $\partial^2 FOC/\partial F \partial t < 0$ for $t < T$. and we find that $\partial FOC/\partial F$ is equal to 0 when $t = T$ and when $t > T$ (the latter is true because F becomes no longer relevant). Thus, we conclude that $\partial FOC/\partial F > 0$ and $\partial d_t/\partial F > 0$ for any $t < T$. The equilibrium technology development effort, d_t^*, is strictly increasing in the imitation cost, F, $\forall\ t < T$.

(ii) Likewise,

$$\begin{aligned} \frac{\partial FOC}{\partial \tau} &= \frac{h_1 \exp[-(\mathcal{L}+t)(\rho+h) - \rho\tau]}{\rho[\rho+h]^2}\{-\delta V\rho[th(\rho+h)-\rho] \\ &\quad - \exp[-(T+\tau)h]\rho[\exp[\rho(t+\tau)]\eta V + \gamma\eta F\rho][\rho - Th(\rho+h)]\} \end{aligned} \tag{a.5}$$

where $h_1 > 0$ and where $\eta \equiv \exp[h(\mathcal{L}+t+\tau)]$. Taking the derivative of the term within the grand brackets with respect to t, we get $\rho(\rho+h)\{-Vh\delta - \exp(\mathcal{L}h)V[\rho - Th\rho - Th^2] < 0$ when $t = T$, because $Th[d_t^*(T), q_t] < 1$ and $\delta > \exp(\mathcal{L}h)$. For the rest of the arguments in (a.5), the negative correlation with t holds as well. Hence, $\partial^2 FOC/\partial\tau\partial t < 0$ when $t = T$. Furthermore, when $t = T$ or when $t > T$, $\partial FOC/\partial\tau = 0$. Thus, $\partial FOC/\partial\tau > 0$ and $\partial d_t^*/\partial\tau > 0$ $\forall t < T$. The equilibrium technology development effort, d_t^*, is strictly increasing in the imitation lag, τ, $\forall t < T$. ■

A.3 Proposition 3: When the innovator maximizes their profit,

(i) d_t^* is strictly increasing in the legal patent length, $\mathcal{L}$, when the imitation cost is relatively high ($\forall\ F > \underline{F}$) and strictly decreasing in the legal patent length, $\mathcal{L}$, when the imitation cost is relatively low ($\forall\ F < \underline{F}$);

(ii) d_t^* has an inverted U-shape with respect to the legal patent length, $\mathcal{L}$, and reaches maximum at $\mathcal{L}_1$;

(iii) the skewness of this curve rises with the imitation lag, τ.

Proof: (i) According to the implicit functional theorem, the sign of $\partial d_t^*/\partial\mathcal{L}$ depends on $\partial FOC/\partial\mathcal{L}$ given that $\partial FOC/\partial d_t^* < 0$.

$$\frac{\partial FOC}{\partial \mathcal{L}} = \frac{\exp[-(\mathcal{L}+t)\rho - \mathcal{L}h]}{\rho}\{h_1[-d_t^* \exp(\mathcal{L}\rho) + \exp(t\rho)(d_t^* + \mathcal{L}V\rho)$$

$$- V\rho T \exp(t\rho + h(\mathcal{L} - T))] + h[(\exp(\mathcal{L}\rho) - \exp(t\rho))(\mathcal{L}d_t^* h_1 - 1)]\}, \quad \text{(a.6)}$$

where T is defined by equation (10). Setting equation (a.6) to 0 and solving respectively for F and $\mathcal{L}$, we get the level of F and $\mathcal{L}$ at which $\partial d_t^*/\partial \mathcal{L} = 0$, denoted as $\underline{F}$ and $\mathcal{L}_1$. Then note that

$$\frac{\partial(\partial FOC/\partial \mathcal{L})}{\partial F} = \frac{\exp[-Th]V[1-hT]}{V + \exp(\mathcal{L}\rho)F\rho} > 0, \quad \text{(a.7)}$$

where $h[d_t^*(t), q_t]T < 1$ because the expected success date evaluated at T holding constant the optimal investment level at t, d_t^*, is weakly later than T (i.e., $T \leq \{1 + h[d_t^*(t), q_t]T\}$ / $\{h[d_t^*(t), q_t] \exp(h[d_t^*(t), q_t]T)\}$). This in turn implies that $\forall\ F > \underline{F}$, $\partial FOC/\partial \mathcal{L} > 0$ and $\partial d_t^*/\partial \mathcal{L} > 0$. Consequently, d_t^* is an increasing function of the length of patent protection $\mathcal{L}$ when the entry barriers are sufficiently high. In contrast, $\forall\ F < \underline{F}$, $\partial FOC/\partial \mathcal{L} < 0$ and thus $\partial d_t^*/\partial \mathcal{L} < 0$. This implies that d_t^* is a decreasing function of the length of patent protection $\mathcal{L}$ when the entry barriers against imitators are sufficiently low.

(ii) Note that when $\mathcal{L} = 0$, T approaches $-\infty$, t and τ are limited to zero, and $\partial FOC/\partial \mathcal{L} = -\exp[-h(T+\tau)]V(T+\tau)h_1$ which approaches $+\infty$. When $\mathcal{L} = T + \tau$, $\partial FOC/\partial \mathcal{L} = \exp[-\rho(T+\tau+t) - (T+\tau)h]/\rho\ \{h_1\ [-\exp[(T+\tau)\rho]d_t^* + \exp(t\rho)\ [d_t^* + (T+\tau)V\rho] - \exp(t\rho + \tau h)\ V\rho T] + [\exp[(T+\tau)\rho] - \exp(t\rho)]\ h[d_t^*(T+\tau)h_1 - 1]\ \} < 0$. Similar to A.1, we find that the FOC and thus d_t^* reach a maximum with respect to $\mathcal{L}$ when $\mathcal{L} = \mathcal{L}_1$, and d_t^* has an inverted U-shape with respect to $\mathcal{L}$.

(iii) Because

$$\frac{\partial FOC}{\partial \mathcal{L} \partial \tau} = \exp[-\mathcal{L}\rho - hT]V[1-hT] > 0, \quad \text{(a.8)}$$

$\underline{F}$ is an decreasing function of τ, and thus the curve that relates d_t^* and $\mathcal{L}$ is thus more skewed to the right (i.e., the range over which $F > \underline{F}$ is enlarged) when τ rises. ■

A.4 Lemma 1: The number of expected imitators, $E_t[z(t_0)]$, is strictly increasing in the innovator's technology development investment, d_t^*.

Proof: The number of expected imitators can be explicitly written as:

$$z(t_0) = \frac{\exp[-\rho(\mathcal{L} + t_0 + \tau][\exp(\mathcal{L}\rho) - \exp(\rho(t_0 + \tau))]V_{t_0}}{F\rho} - 1, \tag{a.9}$$

which is a decreasing function of t_0.

Taking the expectation of (a.9), we get

$$E_t[z(t_0)] = \int_t^T z(t_0)h(d_t, q_t)\exp[-h(d_t, q_t)t_0]dt_0, \tag{a.10}$$

where the density function $f(d_t, q_t, t_0) \equiv h(d_t, q_t)\exp[-h(d_t, q_t)t_0]$ is a strictly increasing function of d_t when $t_0 < (1/h)$ but a decreasing function of d_t when $t_0 \geqslant (1/h)$. A larger d_t^* raises the value of $f(d_t, q_t, t_0)$ for $t_0 < (1/h)$ that has a larger $z(t_0)$ while reducing the value of $f(d_t, q_t, t_0)$ for $t_0 \geqslant (1/h)$ that has a relatively smaller $z(t_0)$. Overall, a larger d_t^* raises the expected number of imitators, $E_t[z(t_0)]$. Hence, $\partial E_t[z(t_0)]/\partial d_t^* > 0$.

Furthermore, in the derived formula for $\partial E_t[z(t_0)]/\partial d_t^*$,we find that when F rises the denominator of $\partial E_t[z(t_0)]/\partial d_t^*$ increases and therefore, $\partial E_t[z(t_0)]/\partial d_t^*$ falls. Thus, the rate at which $E_t[z(t_0)]$ increases in d_t^* is decreasing in F. ■

A.5 Proposition 4: The expected number of imitators, $E_t[z(t_0)]$,

(i) rises with a longer legal patent length, when the imitation cost exceeds a certain level, $\underline{F}$; falls with a longer legal patent length, when the imitation cost is sufficiently lower than $\underline{F}$.

(ii) has an inverted U-shape with respect to the legal patent length, $\mathcal{L}$, when there is imitation, and reaches maximum at $\mathcal{L}_2$ (with $\mathcal{L}_2 < \mathcal{L}_1$).

Proof: Note that

$$\frac{dE_t[z(t_0)]}{d\mathcal{L}} = \frac{\partial E_t[z(t_0)]}{\partial \mathcal{L}} + \frac{\partial E_t[z(t_0)]}{\partial d_t^*}\frac{\partial d_t^*}{\partial \mathcal{L}} \tag{a.11}$$

and that $z(t_0)$ specified in equation (a.9) is an increasing function of $\mathcal{L}$, and T, the upper bound of the support of the integral in equation (a.10), is also an increasing function of $\mathcal{L}$. A larger $\mathcal{L}$ enables to take the increased expectation of (a.9) over a longer interval and raises $E_t[z(t_0)]$. Thus, $\partial E_t[z(t_0)]/\partial \mathcal{L} > 0$. As shown in section A.4, $\partial E_t[z(t_0)]/\partial d_t^* > 0$. According to proposition 3, part (i), when $F > \underline{F}$ (or $\mathcal{L} < \mathcal{L}_1$), $\partial d_t^*/\partial \mathcal{L} > 0$. This indicates that $dE_t[z(t_0)]/d\mathcal{L} > 0$. However, when F is sufficiently lower than $\underline{F}$ (or $\mathcal{L}$ is sufficiently longer than $\mathcal{L}_1$, denoted as $\mathcal{L}_2$) such that the negative effect of the second term ($[\partial E_t[z(t_0)]/\partial d_t^*]$ $[\partial d_t^*/\partial \mathcal{L}] < 0$) outweighs the positive effect of the first term ($\partial E_t[z(t_0)]/\partial \mathcal{L}$), $dE_t[z(t_0)]/d\mathcal{L} < 0$. The intuition behind this result is that, in an extremely competitive industry, the innovator expects more imitators when patents are longer and has fewer incentives to develop the technology.

Similar to A.3, $E_t[z(t_0)]$ attains an interior maximum, and has an inverted U-shape with respect to the legal patent length, $\mathcal{L}$. ■

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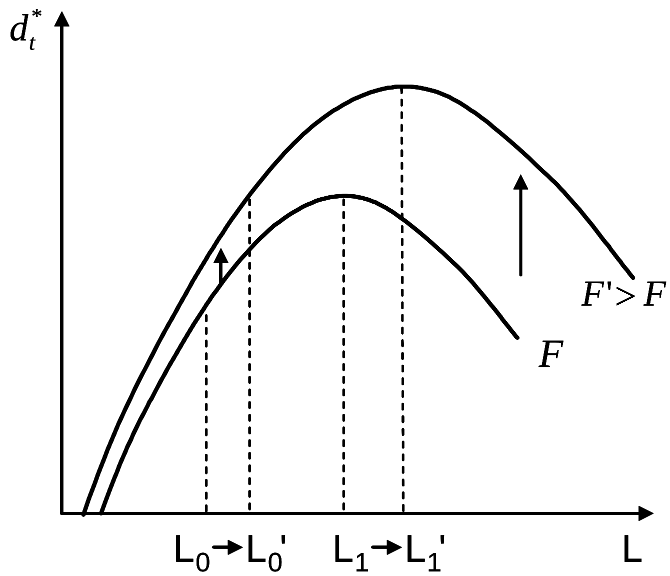

Figure 1: The relationship between legal patent length and optimal technology development investment

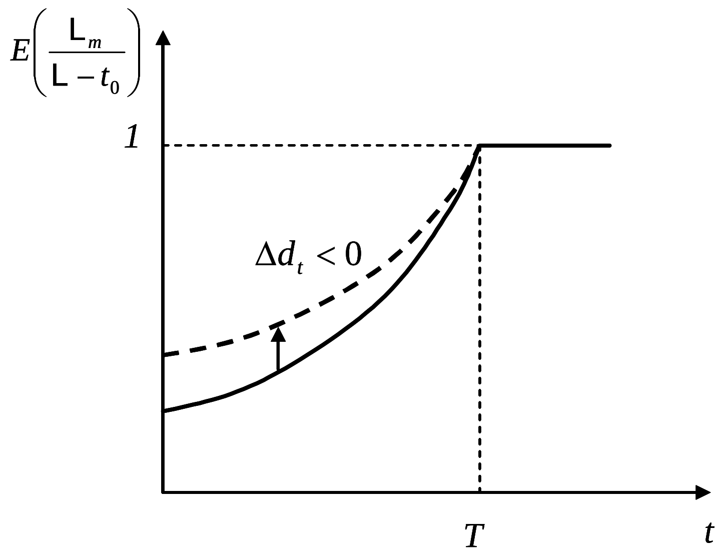

Figure 2.a: The effect of strategic delay on the expected duration of monopoly relative to the remainder of patent length

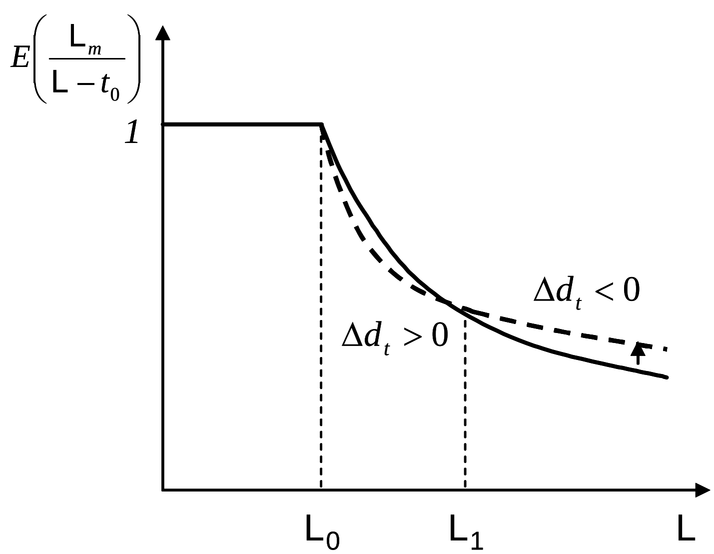

Figure 2.b: The effect of extending patent length on d_t* and the expected duration of monopoly relative to the remainder of patent length

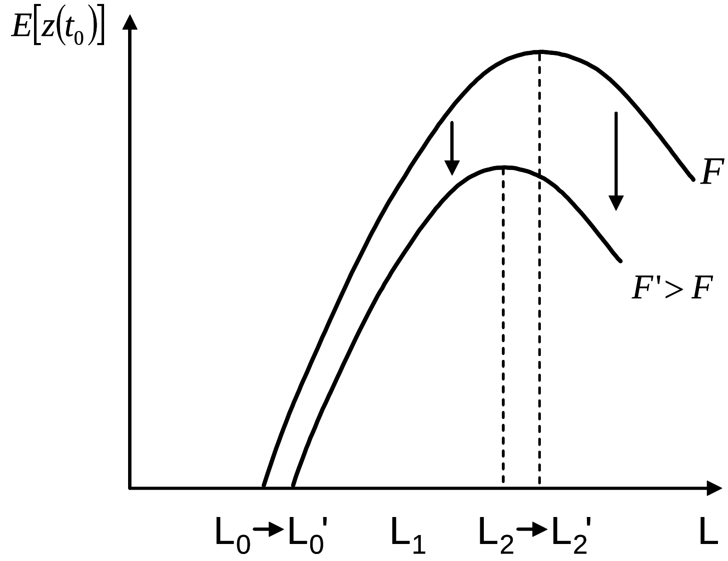

Figure 3: The relationship between legal patent length
and expected number of imitators

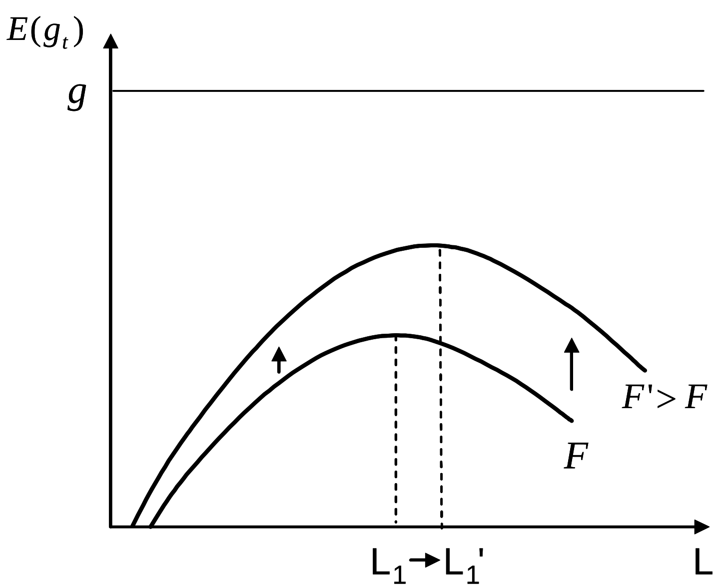

Figure 4: The relationship between legal patent length
and expected rate of technological growth

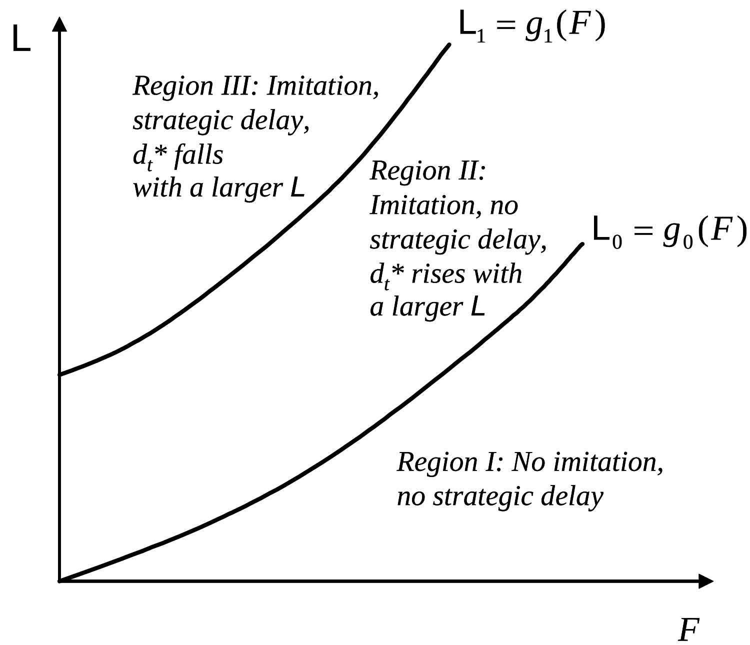

Figure 5: The growth-enhancing patent policy