Marriage Markets and Household Modes of Behavior*

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Abstract

This paper incorporates spousal matching into an intra-household allocation model where spouses endogenously decide whether or not to cooperate in their marriages and individuals' commitment costs vary. When commitment costs are gender symmetric, there is a pure-sorting equilibrium in which some couples play a non-cooperative Nash game (which does not require any spousal commitment) and others act collectively (and with commitment) in determining their household choices. In the pure-sorting equilibrium, the marital gains are shared equally between the husbands and the wives, although the surplus generated by cooperative marriages is higher. When commitment costs are not gender neutral, there can also be mixed-marriage equilibria in which a spouse who is willing to commit marries someone who is not. Since the non-commital spouse can extract from the committed spouse all of the marital material gains of marriage, all mixed couples play the non-cooperative Nash game too. An excess supply of men (women) in a marriage cohort reduces the commitment incentives of men (women) and raises those of women (men). As a corollary, it boosts the number of men (women) in the marriage markets by less than the level implied by the sex ratio. If the gains from marriage fall, not only will fewer individuals marry but also more spouses will choose to enter marriages without a commitment and more couples will act non-cooperatively.

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1 Introduction

A fundamental question in family economics is the degree to which spouses can cooperate within marriage. While there exists no current consensus on this issue, whether marriage can be characterized as a fully cooperative institution or one in which members bargain over resource allocation without commitment to cooperate has profound implications–both theoretical and empirical.

The traditional theoretical approach to analyze household choices takes the family as the relevant decision-making unit.[1] The non-unitary household models provide an alternative to this approach by treating the individual members of the family as the core decision-makers. Starting in the early 1990s, the empirical literature began to support the notion that relative spousal incomes matter for family decisions and intra-household allocations.[2] Consequently, the non-unitary household models have emerged as the compelling theoretical alternative for analyzing the economics of the family.

There are three classes of non-unitary models in the existing literature: First, we have the 'collective models' where household members act with commitment and cooperation to choose from a host of Pareto efficient choices. The generalized underpinning of this model was provided by Becker (1981) and Chiappori (1988, 1992). A closely related strand involves the 'cooperative bargaining models' in which a cooperative process–typically, although not exclusively, the Nash bargaining paradigm–determines household allocations. The seminal examples in this category include Manser and Brown (1980), MacElroy and Horney (1981), and Sen (1983). And third, we have models of 'non-cooperative Nash bargaining' where household members choose their actions taking as given those of other family members. Some examples in this line are Lundberg and Pollak (1993), Chen and Woolley (2001), and Basu (2006).

The collective models as well as the cooperative bargaining theories depend on the Pareto optimality of household decisions and their testable predictions and identification power rest on whether or not household members can cooperate to generate those

[1]The theoretical foundations of this literature is provided by two seminal papers. In Samuelson (1956), a consensus approach is emphasized as the rationale for treating the household allocation problem as that of maximizing a single household utility function. In Becker (1981), the existence of an altruistic household member is shown to generate outcomes that maximize total family income even in the presence of family members with divergent preferences.

[2]See, for example, Browning et al. (1994), Lundberg et al. (1997), Chiappori et al. (2002), and Udry (1996).

efficient outcomes. As Chiappori (1988, 1992) shows Pareto optimality enables one to recover the underlying preference structure of household members as well as the implicit sharing rule that influences the intra-household allocations among different individuals.

When spousal choices such as labor supply and production specialization influence not only household income but allocations within it, cooperative behavior would be harder to sustain because it could be costly for household members to commit to efficient choices. Indeed, there are empirical findings which suggest that spousal specialization and labor force detachment influence the relevant threat points. For example, married men work longer hours in the market and have substantially higher wages than unmarried men, and married women work less and have lower wages compared to single women. Together these findings imply that wives who commit most or all of their time to domestic production could be worse off in divorce whereas husbands who work full time could be better off.[3] Furthermore, recent time-use statistics reveal that women work significantly longer hours at home than men, even though their hours worked in the labor market have converged in the last three decades.[4]

When spousal choices involve costly commitment and, hence, they affect the household balance of power, one can no longer restrict attention to the efficient frontier. Since commitment to cooperate is potentially costly and it introduces a hold-up problem, repeated interactions and an appeal to the Folk theorem would not help to restore efficiency either. In such cases, the plausible alternative is to model spousal behavior as a non-cooperative bargaining process that can stray from efficient outcomes.[5] The major drawback of this approach is that household allocations are likely to be inefficient which in turn makes it impossible to recover the preferences of household members and the sharing rules that determine intra-household allocations.

Empirical studies of household behavior too have to deal with the fact that spousal labor supply choices can influence the household balance of power. One less-than-ideal solution to this is to acknowledge that labor income can influence household allocations but then to constrain labor supply choices and assume separability between consumption and leisure.

In short, the choice of modelling continues to plague theorists and empiricists alike

[3] See Gronau (1986), Daniel (1992), and Korenman and Neumark (1992). For a detailed overview, see Weiss (1997).

[4] For details, refer to Aguiar and Hurst (2006).

[5] For examples of such models, see Lundberg and Pollak (1994, 2003), Rasul (2005), and Iyigun and Walsh (2007a).

and the existing methodological approach consists of making an assumption about the efficiency of household choices, then utilizing theoretical and empirical models consistent with that assumption. But while we lack a theoretical underpinning of marital commitment versus non-cooperative bargaining, recent empirical work has made attempts to identify the valid mode of intra-household actions. For example, Del Boca and Flinn (2005) estimate a structural model and find some weak evidence in favor of the collective mode of behavior among couples in the United States. In contrast, Mazzocco (forthcoming) presents an empirical test of intra-household commitment and, using PSID data, rejects the hypothesis that household members can commit to future allocations of resources.

This paper is intended to fill the theoretical gap. The object of it is a prototype model in which both cooperative and non-cooperative modes of household behavior are sustained among heterogenous couples and the general equilibrium price of spousal commitment, which is derived endogenously, helps to determine which mode of action is chosen by any particular couple. In the model, couples match in the marriage markets, individuals vary according to their costs of commitment to cooperate in marriage, and all intra-household allocations are determined endogenously. The marital surplus of two committed spouses is higher than that of the marriage of two uncommitted partners. Those spouses who can commit to cooperate are willing to incur a cost for doing so and they abide by the ex-ante spousal allocations determined in the marriage markets. As a result, when two committed partners marry, they abide by efficient household choices and allocations. In contrast, when two uncommitted partners marry, they play a non-cooperative Nash game and the choices such couples make yield inefficient outcomes in the conventional sense. Furthermore, when a non-committed individual marries a committed spouse, the former can take advantage of his/her partner's decision and extract all of the surplus generated by their marriage. The essential idea here is that a committed spouse makes a costly–and, perhaps, marriage-specific investment–which leaves him/her vulnerable to opportunistic spousal behavior. Thus, all couples who disagree abut marital commitment resort to the non-cooperative Nash bargaining game as well. I investigate the rational-expectations equilibrium that arises under such circumstances.

The main findings that emerge from the model are as follows: When marriage preferences and commitment costs are gender symmetric, the equilibrium involves pure sorting where some couples do not cooperate and some do. In such an equilibrium, the marital gains are shared equally between the husbands and the wives, although the surplus generated by cooperative marriages is higher. When marriage preferences or commitment costs are not gender neutral, however, there may also be mixed marriages in which a spouse who is not willing to commit will be matched with a partner who is willing to do so. In such marriages, the uncommitted spouse can extract all of the marital gains from the marriage as long as his or her partner stays committed. Consequently, all mixed couples also resort to Nash bargaining to determine their intra-marital choices.

The important implication of this is that when there is a mixed-marriage equilibrium with, say, committed men in short supply, men's commitment incentives will be higher because committed women compete more intensely for committed men; as a result, all women get a lower return from marriage and all men get a higher return. In general, an excess supply of men (women) in a marriage cohort reduces the commitment incentives of men (women) and raises those of women (men). As a corollary, it boosts the number of men (women) in the marriage markets by less than the level implied by the sex ratio.

If the gains from marriage fall–for example, due to technological change which diminishes the returns to scale from cohabitation, a la Greenwood, Seshadri and Yorukoglu, 2005–then not only will fewer individuals marry, but also more spouses will choose to enter marriages without commitment and act non-cooperatively once they marry.

In addition to the papers reviewed above, this paper shares some similarities with a burgeoning strand in the economics of the family literature which incorporates some aspect of pre-marital decision-making (such as educational attainment) and spousal matching into a model of intra-household decision-making. Recent examples include Peters and Siow (2002), Browning et al. (2003), Chiappori et al. (2006) and Iyigun and Walsh (2007b).

The remainder of this paper is organized as follows: In section 2, I present the generic model. In Section 3, I discuss a specific example in which couples can cooperate by specializing in home production and labor market work. In Section 4, I conclude.

2 The Basic Model

2.1 Assumptions

The populations of men and women are large and equal in mass.[6] Men and women are completely identical in their preferences and opportunities and they all live for one period. At the beginning of the period, individuals decide whether they want to get married or stay single. Those who choose to get married then match with their mates. Competition over mates determines who marries whom. These assignments, together with the known individual characteristics described below, guides the individuals' decisions to marry and, if they choose to do so, whether they can commit to cooperate in their marriages.

2.2 Household Behavior Modes

I denote a particular man by i and a particular woman by j. All individuals of a given gender produce the same material output when they are single, but singles' output level may differ by gender; I denote the material utility of a single man i by ζ_m and that of a single woman j by ζ_w.

Definition 1 *A couple* $\{i,\, j\}$ *takes the actions* $a_i,\, a_j \in [0,\, 1]$ *to produce the* joint material output *given by* $\zeta_{ij} = \zeta(a_i,\, a_j)$*. The material output* $\zeta_{ij} = \zeta(a_i,\, a_j)$ *is twice differentiable and concave in* a_i *and* a_j *with* $\{a_i^c,\, a_j^c\} = \arg\max\, \zeta(a_i,\, a_j)$.

The couple's output $\zeta_{ij} = \zeta(a_i,\, a_j)$ can be divided between the spouses and the utility of each partner is linear in the share he\she receives. This reflects transferable utility between the spouses. The *material surplus* of the marriage is then defined as

$$z_{ij} \equiv \zeta(a_i, a_j) - \zeta_m - \zeta_w. \tag{1}$$

There are two possible modes of equilibrium household behavior regarding how a_i and a_j are chosen. In particular, the actions a_i and a_j can be taken cooperatively and with spousal commitment or they can be made non-cooperatively and without

[6] I address the impact of the sex ratio in Section 2.6 below.

commitment.[7] Let the type of man i be represented by $I(i)$, where $I(i) = c$ if i is willing to make a potentially costly commitment to his marriage and $I(i) = n$ if he is not, and the type of woman j be denoted by $J(j)$, where $J(j) = c$ if j is willing to commit and $J(j) = n$ if she is not. Let U_j and V_i respectively denote the material allocations of wife j and husband i respectively. We can now define the household modes of action as follows:

Definition 2 ***(Cooperation and Commitment):*** *When both spouses commit to cooperate, they maximize the joint material output of their marriage and abide by the ex-ante spousal allocations that are determined in the marriage markets. Hence,* $\zeta_{ij} = \zeta_{cc} \equiv \zeta(a_i^c, a_j^c)$ *and* $U_j = U_c$, $V_i = V_c$, *where* U_c *and* V_c *represent the endogenously determined shadow prices of committed husbands and committed wives.*

Thus, couples' cooperate by implementing the actions a_i^c and a_j^c which generate efficiency in marriage. The mechanism by which they implement those actions is adherence to the ex-ante spousal allocations determined in the marriage market.

Definition 3 ***(Non-cooperative Nash Bargaining):*** *When neither spouse can credibly commit to maximizing the joint material output of their marriage, an uncommitted husband* i *married to an uncommitted wife* j *solves*

$$\max_{a_i} V_n(a_i, a_j^n) = \max_{a_i}[\zeta(a_i, a_j^n) - \zeta_m - \zeta_w - U_j(a_i, a_j^n)] \qquad s.t. \quad U_n + V_n \ \leq z_{nn} \ , \tag{1.a}$$

and an uncommitted wife j *married to an uncommitted husband* i *solves*

$$\max_{a_j} U_n(a_i^n, a_j) = \max_{a_i}[\zeta(a_i^n, a_j) - \zeta_m - \zeta_w - V_i(a_i^n, a_j)] \qquad s.t. \quad U_n + V_n \ \leq z_{nn} \ , \tag{1.b}$$

where, by definition, a_i^n *represents the Nash best response to* a_j^n *and vice versa. The Nash-bargained material output* $\zeta(a_i^n, a_j^n) \equiv \zeta_{nn}$ *is such that,* $\forall\ a_i^n \neq a_i^c$, *and* $a_j^n \neq a_j^c$, $\zeta_{nn} < \zeta_{cc}$ *and* $a_i^n = \arg\max V_n(a_i, a_j^N) \equiv V^N$ *and* $a_j^n = \arg\max U_n(a_i^n, a_j) \equiv U^N$.

[7] As I noted earlier, action a could involve an optimal household role or an efficient level of market labor supply. See the example in Section 3 below, which covers the case of endogenous spousal specialization and labor time allocations commensurate with it as the specific actions taken in marriage which impact the efficiency of household choices.

To wrap things up, consider the outcome when an uncommitted individual marries a committed spouse. In such a mixed marriage, the committed spouse could willingly choose the action that maximizes the marital material surplus (a_i^c or a_j^c) while the uncommitted partner, capitalizing on his or her spouses' commitment, chooses to maximize his or her material utility subject to the committed partner's marital participation constraint. Let the behavior of uncommitted partners be defined as "opportunistic" when they are paired up with committed spouses, with a_i^o and a_j^o representing the actions commensurate with "opportunistic" mode of action by man i and woman j. Letting the material output of a committed husband i and an opportunistic wife j equal $\zeta_{cn} \equiv \zeta(a_i^c, a_j^o)$ and that of an opportunistic husband i and a committed wife j be defined as $\zeta_{nc} \equiv \zeta(a_i^o, a_j^c)$, we introduce the following definition:

Definition 4 ***(Opportunistic Mixed-Marriage Mode):*** *When husband i is not committed to the efficient choices in his marriage but wife j is,*

$$V_i(a_i^c, a_j^o) = \zeta_{nc} - \zeta_m - \zeta_w \equiv z_{nc} \qquad \text{and} \qquad U_j(a_i^o, a_j^c) = 0 \tag{1.c}$$

and when husband i is committed but wife j is not,

$$U_j(a_i^c, a_j^o) = \zeta_{cn} - \zeta_m - \zeta_w \equiv z_{nc} \qquad \text{and} \qquad V_i(a_i^c, a_j^o) = 0 \ . \tag{1.b}$$

In words, when they are married to a committed spouse, uncommitted individuals act opportunistically and extract all of their marital surplus. This leads to the following important observation:

Lemma 5 *In equilibrium, no spouse can behave opportunistically. Mixed-couples resort to Nash bargaining and all spouses in such marriages receive U^N and V^N.*
Proof. *$\forall\ I(i) = c \wedge J(j) = n$, $V_i(a_i^c, a_j^o) = 0 < V^N$ and $\forall\ I(i) = n \wedge J(j) = c$, $U_i(a_i^o, a_j^c) = 0 < U^N$. Thus, $\forall\ I(i) = c \wedge J(j) = n$, $a_i^n = \arg\max V_i(a_i, a_j^o)$. Likewise, $\forall\ I(i) = n \wedge J(j) = c$, $a_j^n = \arg\max U_j(a_i^o, a_j)$. As a result, $\forall\ I(i) = c \wedge J(j) = n$, $U_j = U^N \wedge V_i = V^N$. And, $\forall\ I(i) = n \wedge J(j) = c$, $U_j = U^N \wedge V_i = V^N$.* ■

Consequently, when a committed man (or woman) marries an uncommitted woman (or man), they generate the output $\zeta_{cn} = \zeta_{nc} = \zeta_{nn}$ and the surplus $z_{cn} = z_{nc} = z_{nn}$. Since cooperation yields a Pareto efficient outcome for all couples who can commit and cooperate, it follows that

$$z_{cc} + z_{nn} > z_{nc} + z_{cn} \; = \; 2z_{nn} \; . \tag{2}$$

Hence, marital surplus rises with the commitment level of both partners.

Commitment to a marriage (and, by extension, a spouse) is costly. It involves idiosyncratic non-pecuniary costs (benefits) denoted by μ_i for men and μ_j for women. Two common examples such costly commitment are (i) specialization within the household by market and non-market time use; and (ii) the decision to have children, which might require differential time involvement by gender and spouse. Note, however, an important distinction that I make here: typically, all choices that involve marital public goods or spousal specialization influence the material resources of the household and such choices might involve personal material costs to each spouse as well. For example, having children might require one spouse to withdraw from the labor market and a prolonged period of labor force detachment could hurt his/her future wages (Korenman and Neumark, 1992). Clearly, all such material costs are embedded in the couples' material output discussed above. As such, the non-material cost of commitment, μ, represents the additional utility costs to the person of marital commitment. The idea is that, due to innate differences or family backgrounds, some men and women find it more easy to commit to marital decisions and choices than others–whatever the material cost of such choices might be (as reflected in the couples' material production).

In addition, individuals derive non-material gains from marriage which I denote by θ_i and θ_j for man i and woman j. These idiosyncratic cost and preference parameters are assumed to be independent of each other and across individuals. I denote the distributions of θ and μ by $F(\theta)$ and $G(\mu)$ for men and by $\hat{F}(\theta)$ and $\hat{G}(\mu)$ for women, respectively. All four distributions are symmetric around their means.[8]

[8] By construction, the individual costs and benefits are observable at the time of marriage to both spouses. If the costs and benefits are not directly observable, then the results discussed below would still go through unaltered if individuals act on the basis of noisy but unbiased proxies for them. If, instead, the decision to commit is made on the basis of expected costs and benefits, the main results would still attain but the derivations would become cumbersome.

We can now define the total *marital surplus* (including the material and non-material components) generated by the marriage of i and j as

$$s_{ij} = \begin{cases} z_{nn} + \theta_i + \theta_j & \text{if either } i \text{ or } j \text{ is not committed,} \\ \\ z_{cc} + \theta_i + \theta_j - \mu_i - \mu_j & \text{if both } i \text{ and } j \text{ are committed.} \end{cases} \tag{3}$$

2.3 The Marriage Market

Even though uncommitted and mixed couples rely on Nash bargaining, spousal shadow prices (i.e., the ex-ante intra-marital allocations derived in the marriage market) still play a key role because not only committed couples rely on them for their intra-family choices, but also those shadow prices reflect the imbalance in the supply of committed men relative to women (which in turn alters the incentives for marital commitment). So as a first order of business, we need to identify how spousal shadow prices are determined.

Spousal Assignments: Any *stable* assignment of men to women must maximize the *aggregate surplus* over all possible assignments (Shapley and Shubik, 1972). The dual of this linear programming problem posits the existence of non-negative shadow prices associated with the constraints of the primal that each person can be either single or married to one spouse. Accordingly, the complementarity slackness conditions require

$$s_{ij} \ \leq \ v_i \ + \ u_j \ . \tag{4}$$

Condition (4) yields

$$v_i = \max\{\max_j (s_{ij} - u_j)\,,\ 0\} \qquad \text{and} \qquad u_j = \max\{\max_i (s_{ij} - v_i)\,,\ 0\}, \tag{5}$$

which means that the assignment problem can be *decentralized*. That is, given the shadow prices u_j and v_i, each agent marries a spouse that yields the highest "profit" or remains single. Alternatively, we can view the shadow prices u_j and v_i as the reservation utility levels that woman j and man i require to participate in *any marriage*.

With this specification, we have a convenient structure in which the interactions between agents depend on their commitment type only. In particular, we can write the endogenously-determined shadow prices of married man i and woman j in the following forms:

$$v_i = \max\left(\hat{V}_i + \theta_i,\ 0\right) \quad \text{and} \quad u_j = \max\left(\hat{U}_j + \theta_j,\ 0\right), \tag{6}$$

where

$$\hat{V}_i \equiv \begin{cases} V_I & \text{if } I(i) = n, \\ V_I - \mu_i & \text{if } I(i) = c, \end{cases}$$
$$\hat{U}_j \equiv \begin{cases} U_J & \text{if } J(j) = n, \\ U_J - \mu_j & \text{if } J(j) = c, \end{cases} \tag{6.a}$$

and where, given Definitions 2 and 3, V_I and U_J are the *shares* that the partners receive from the material surplus of the marriage (not accounting for the idiosyncratic effects θ_i and θ_j and the costs of commitment μ_i and μ_j). Assuming that at least one person in each class is married, this implies that

$$V_I = \max_J [z_{IJ} - U_J] \qquad \text{and} \qquad U_J = \max_I [z_{IJ} - V_I]\ . \tag{7}$$

All agents receive the *same* share of the material surplus z_{IJ} no matter whom they marry.[9] Any man (woman) with a given pair of commitment cost and non-economic marital gain can be classified as a given "type", and all those individuals whose costs are sufficiently low will be committed. No committed individual who asks for a higher share than the "going rate" can obtain it because he (she) can be replaced by an equivalent alternative.

Stability Conditions: Although there are equal numbers of men and women in total, it is possible that the equilibrium numbers of married men and women who are committed will differ. Hereafter, I assume that there are some committed men who marry committed women and some uncommitted men who marry uncommitted

[9]For committed couples, this is because all committed individuals on the other side rank them in the same manner. For uncommitted couples, this is not generally be true but, due to the fact that the Nash bargaining game in this model yields identical payoffs conditional on one's gender, it is also true among uncommitted couples.

women. This, together with the fact that uncommitted couples' shares are determined via Nash bargaining, means that the equilibrium shares satisfy

$$U_c + V_c = z_{cc}, \tag{8}$$

$$U_n + V_n = z_{nn} \qquad \text{with} \qquad U_n = U^N \qquad \text{and} \qquad V_n = V^N \ , \tag{9}$$

where U^N and V^N represent the Nash bargaining outcomes as they are described in Definition 2.

We can now classify the possible matching patterns as follows: Under *pure* assortative mating, committed men marry only committed women and uncommitted men marry only uncommitted women. Consider the two additional restrictions:

$$U_n + V_c \geq z_{cn} = z_{nn} \qquad \text{and} \qquad V_c = V^N, \tag{10}$$

$$U_c + V_n \geq z_{nc} = z_{nn} \qquad \text{and} \qquad U_c = U^N \ . \tag{11}$$

If, among the married, there are more men who are willing to commit to their marriages than women, some committed men will marry uncommitted women and condition (10) will hold with the weak inequality in (10) being satisfied as an equality. If there are more committed women than men among the married, then (11) will apply with the weak inequality in (11) holding as an equality. It is impossible that both conditions will hold as equalities because, together with (8) and (9), this would imply

$$z_{cc} + z_{nn} = z_{nc} + z_{cn} \ = \ 2z_{nn} \qquad \Rightarrow \qquad z_{cc} \ = \ z_{nn} \tag{12}$$

which violates assumption (2) that the commitment levels of the spouses are complements, i.e., $z_{cc} > z_{nn}$. Thus, either committed men marry uncommitted women or committed women marry uncommitted men but not both.

When types mix and there are more committed men than committed women among the married, there are some marriages between uncommitted women and committed men in equilibrium. In those marriages, couples resort to the Nash game so that equations (8), (9) and (10) applying as a strict equality generate

$$U_c - U_n = z_{cc} - z_{nn}, \qquad \text{and} \qquad V_c - V_n = 0 \ . \tag{13}$$

In words, when uncommitted women marry committed men, then the former can take advantage of the latter to extract all of the material surplus. As a result, some men who would otherwise be willing to commit to their marriages would be forced to resort to Nash bargaining in their marriages to uncommitted women. Since some committed men are still lucky enough to marry committed women, men compete for committed women up to the point where $V_c = V_n = V^N$. This in turn generates the maximum marital return for committed women which equals $U_c = z_{cc} - z_{nn}$.

If there are more committed women then men among the married, then there are some marriages between uncommitted men and committed women in equilibrium. In those marriages, $U_c = U_n = U^N$. Together with conditions (8), (9) and (11) holding as a strict equality, this yields

$$V_c - V_n = z_{cc} - z_{nn}, \qquad \text{and} \qquad U_c - U_n = 0 \; . \tag{14}$$

Thus, when there are more committed women than men in the marriage market, committed men extract all of their marital surplus.

The main point here is that the differences $U_c - U_n$ and $V_c - V_n$ represent the *return to marital commitment* for women and men, respectively. Consider the case of women, for instance: The quantity $z_{cc} - z_{nn}$, which reflects the return a committed woman earns in the marriage market when her type is in short supply, defines the *upper bound* on the return to commitment in marriage, whereas her return which vanishes when her type is on the long side of the market defines the *lower bound*. The essential observation is that women receive their upper bound in mixed equilibria in which there are more committed men than women among the married and they receive their lower bound in mixed equilibria in which there are more committed women than men among the married.

Proposition 6 *(a) In the pure-sorting, fully symmetric equilibrium,* $U_c = V_c = z_{cc}/2$ *and* $U_n = V_n = z_{nn}/2$ *with* $U_c = V_c > U_n = V_n$*; (b) In the mixed-marriage equilibrium with committed men in short supply,* $V_c = V_n = V^N$*,* $U_n = U^N$ *and* $U_c = z_{cc} - V^N$ *with* $U_c > V_c$*; (c) In the mixed-marriage market equilibrium with committed women in short supply,* $U_c = U_n = U^N$*,* $V_n = V^N$ *and* $V_c = z_{cc} - U^N$ *with* $V_c > U_c$*.*
Proof. *(a) When the model is completely symmetric,* $U^N = V^N$*, and* $\forall\theta$*,* $F(\theta) = \hat{F}(\theta)$*,* $\forall\mu$*,* $G(\mu) = \hat{G}(\mu)$*. Then, only (8) and (9) hold as equalities and (5) yields equal shares for all spouses; (b) Follows directly from (8), (9), and (11) holding as a strict equality; (c) Follows directly from (8), (9), and (10) holding as a strict equality.*
■

An important issue is whether some of the "gross material shares" defined above, U_J and V_I, can be non-positive in equilibrium. In particular, if couples can exchange and transfer "signs of endearment", then the material shares can be negative in equilibrium when the non-material utility from marriage is relatively high. But note that the Nash bargaining outcome rules out non-negative material spousal allocations. That is, in all equilibria, $U_n = U^N > 0$ and $V_n = V^N > 0$. In addition, I prove in Appendix section 5.1 that $U_c \geqslant U_n$ and $V_c \geqslant V_n$. Hence, all material equilibrium allocations in this model are strictly positive.

2.4 Optimal Decision-making Modes

I assume rational expectations so that, in equilibrium, individuals know V_I and U_J which, together with the costs of commitment and the non-economic gains of marriage, are sufficient statistics for the individuals decision to commit to the ex-ante spousal allocations or try to renege on them once they marry. Given these shares, the knowledge of their own idiosyncratic preferences for marriage, θ, and costs of commitment, μ, agents know for sure whether they will marry, and if they do marry, whether they will credibly commit to the ex-ante expected spousal allocations, V_c and U_c, dictated by the marriage market.

In particular, man i chooses to credibly commit to cooperate in marriage if

$$\zeta_m + \max(V_c + \theta_i - \mu_i,\ 0)\ >\ \zeta_m + \max(V_n + \theta_i, 0). \tag{15}$$

Similarly, woman j chooses to commit if

$$\zeta_w + \max(U_c + \theta_j - \mu_j,\ 0)\ >\ \zeta_w + \max(U_n + \theta_j, 0). \tag{16}$$

The LHS of the inequalities in (15) and (16) represent the total utility individuals i and j get when they are committed in marriage and the RHS of the two equations represent their utility when they are not.

Proposition 7 *(1.a) Men for whom $\theta < -V_n$ do not marry without making a credible commitment; (1.b) Men for whom $-V_n < \theta < \mu - V_c$ marry but do not make a commitment; (1.c) Men for whom $\theta \geqslant \mu - V_c$ marry and are able make a credible commitment. (2.a) Women for whom $\theta < -U_n$ do not marry without making a credible commitment; (2.b) Women for whom $-U_n < \theta < \mu - U_c$ marry but do not make a commitment; (2.c) Women for whom $\theta \geqslant \mu - U_c$ marry and are able make a credible commitment.*

Proof. *Appendix 5.1 shows* $V_n \leq V_c$ *and* $U_n \leq U_c$*. Thus, men with* $\theta < \min(-V_n, \mu - V_c)$ *stay single; those with* $\mu - V_c > \theta > -V_n$ *marry and play a non-cooperative Nash game once they do; and those with* $\theta > \max(-V_n, \mu - V_c)$ *always marry. The latter suggests men for whom* μ *is such that* $-V_n \geqslant \mu - V_c$ *marry and choose the non-cooperative Nash game once they do. Same arguments analogously hold for women.* ■

For the remainder of the analysis, I assume that the variability in θ and μ are large enough to ensure that *all* three regions are always non-empty in equilibrium. In particular, I assume that there are some men and women who prefer not to marry. That is, $\theta_{\min} < \min(-V_n, \mu_{\max} - V_c)$ and $\theta_{\min} < \min(-U_n, \mu_{\max} - U_c)$. I also assume that there are some men and women who would always marry so that $\theta_{\max} > \max(-V_n, \mu_{\max} - V_c)$ and $\theta_{\max} > \max(-U_n, \mu_{\max} - U_c)$. Finally, with $\mu_{\min} < V_c, U_c$ and $\mu_{\max} > V_c, U_c$, I ensure that there are always some men and women who marry and credibly commit as well as some men and women who marry but cannot commit.

Figure 1 describes the choices made by different men, taking as given those made by women. The upward-sloping bold line represents $\theta = \mu - V_c$, which delineates the region of commitment in marriage from staying single and marrying without commitment. The vertical bold line in the second quadrant depicts $\theta = -V_n$ which represents the threshold for staying single and marrying without commitment. All combinations of θ and μ to the right of the upward-sloping line represent men who marry with commitment; all those to the left of the upward-sloping and vertical lines apply to men who choose to stay single; and all pairs of θ and μ to the right of the vertical line and left of the upward-sloping line represent those men who want to marry but cannot commit.

[Figure 1 about here.]

Given equations (15) and (16) and the distributions of θ and μ, the proportion of men who marry and credibly commit to cooperate is

$$\int_{-\infty}^{\infty} G(V_c + \theta) f(\theta) d\theta \;, \tag{17}$$

and the proportion of all men who marry is

$$[1 - F(-V_n)] \; + \int_{-\infty}^{-V_n} G(V_c + \theta) f(\theta) d\theta \; . \tag{18}$$

The first term in equation (18) includes all men who marry and cannot commit as well as some of those who marry and commit, i.e., those men who possess $-V_n \leq \theta$ get married regardless of the value of their θ's. The second term in that equation covers those who get married only by making a commitment i.e., those men who possess $-V_n > \theta \geqslant \mu - V_c$.

Obviously, the higher is the gross return to commitment in marriage, V_c, the higher is proportion of men who marry and commit. Nonetheless, a common increase in the levels V_c and V_n, which would not alter men's net return from commitment, also raises the likelihoods of marriage and commitment because it makes marriage more attractive relative to singlehood and commitment obtains its return only within marriage. Analogous expressions hold for women too.

2.5 Equilibrium

If the equilibrium involves pure sorting on the basis of spousal commitment, then the numbers of men and women who *marry* and credibly *commit* will be the same. Using condition (17), we can derive this condition as

$$\int_{-\infty}^{\infty} G(V_c + \theta) f(\theta) d\theta \; = \; \int_{-\infty}^{\infty} \hat{G}(U_c + \theta) \hat{f}(\theta) d\theta. \tag{19}$$

Together with condition (8), (19) yields a system of two equations in two unknowns. That is, (8) and (19) yield unique solutions for V_c and U_c. If a feasible solution exists, then there will be equal numbers of men and women in the marriage markets who can commit. As a result, we will have a pure sorting equilibrium in which there are some couples who commit to the ex-ante agreed upon allocations and some couples who do not. Once married, the latter type of couples play a Nash game to determine their household allocations, while the former make household decisions in a fully cooperative manner.

If there is some mixing of types, equation (19) is replaced by an inequality and the shares are determined by the boundary conditions on the returns to marital commitment for either men or women, whichever is applicable. If there are more committed men than women among the married, then (19) evaluated at $U_c = z_{cc} - V^N$ and $V_c = V^N$, will produce

$$\int_{-\infty}^{\infty} G(V^N + \theta) f(\theta) d\theta \; > \; \int_{-\infty}^{\infty} \hat{G}(z_{cc} - V^N + \theta) \hat{f}(\theta) d\theta \tag{19.a}$$

As a result, we will have a mixed equilibrium in which some committed men marry uncommitted women.

Conversely, if there are more committed women than men among the married, (19) evaluated at $V_c = z_{cc} - U^N$ and $U_c = U^N$, yields

$$\int_{-\infty}^{\infty} G(z_{cc} - U^N + \theta) f(\theta) d\theta \; < \; \int_{-\infty}^{\infty} \hat{G}(U^N + \theta) \hat{f}(\theta) d\theta \tag{19.b}$$

and we will have a mixed equilibrium in which, in addition to the marriages of purely committed and uncommitted spouses, we will also have some pairings among whom men who are willing to commit are married to women who are not.

By adding the numbers of men and women who want to marry without commitment to both sides of (19), we can derive the aggregate marriage market clearing condition:

$$F(V^N) \; + \; \int_{-\infty}^{-V_n} G(V_c + \theta) f(\theta) d\theta \; = \; \hat{F}(U^N) \; + \; \int_{-\infty}^{-U_n} \hat{G}(U_c + \theta) \hat{f}(\theta) d\theta \; . \tag{20}$$

Equation (20) may or may not hold even if (19) holds and there are equal numbers of committed men and women in the marriage market. The reason is that there are two types of individuals in the marriage market on both sides and the price of only committed spouses is influenced by changes in demand and supply. That is, the Nash bargaining payoffs, U^N and V^N, are invariant to changes in market conditions. Thus, there is no guarantee that this condition holds in a pure-sorting equilibrium unless the model is completely symmetric so that, $U^N = V^N$, and $\forall \theta$, $F(\theta) = \hat{F}(\theta)$, $\forall \mu$, $G(\mu) = \hat{G}(\mu)$, which together yield $V_c = U_c = z_{cc}/2$ and $U^N = V^N = z_{nn}/2$. In fact, when the Nash bargaining payoffs are not symmetric so that $U^N \neq V^N$, (20) will not hold in a pure-sorting equilibrium even if $\forall \theta$, $F(\theta) = \hat{F}(\theta)$, $\forall \mu$, $G(\mu) = \hat{G}(\mu)$. And in a mixed equilibrium (20) will definitely not hold.

Whether the equilibrium involves pure sorting or is mixed depends on whether or not equation (19) holds. When (19) holds, then we have a pure-sorting equilibrium but given that (20) does not necessarily hold there can be two kinds of pure-sorting equilibrium: in one, (20) holds so that the marriage market clears and all spouses find a partner. In the other, (20) does not hold as a result of which some individuals who want to marry without commitment cannot find a spouse.

The two types of solutions (pure and mixed) are illustrated in Figures 2.a through 4. In all of the figures, I depict the equilibrium conditions given by (19) and (20) in terms of V_n and V_c after eliminating U_n and U_c using (8) and (9). The downward-sloping line defines equation (20); it represents the combinations of V_n and V_c that maintain equality in the numbers of men and women who wish to marry, regardless of whether they are also willing to commit to cooperate in marriage. The horizontal line represents equation (19); it describes the level of V_c that maintains equality in the numbers of men and women who wish to marry *and* commit. The slopes of these lines are determined as follows: An increase in V_n (and a reduction in U_n), keeping V_c and U_c constant, induces more men and fewer women to prefer marriage. An increase in V_c (and a reduction in U_c) holding V_n and U_n has a similar effect. Thus, V_n and V_c are substitutes in terms of their impact on the incentives of men to marry and U_n and U_c are substitutes in terms of their impact on the incentives of women to marry. Therefore, equality in the number of men and women who wish to marry can be maintained only if V_c declines when V_n rises.[10] At the same time, an increase in V_c (and a reduction in U_c), regardless of what happens with V_n and U_n, increases the number of men that would marry and credibly commit to cooperate, while it reduces the number of women who can do the same. Therefore, given the cumulative distributions $F(\theta)$, $\hat{F}(\theta)$, $G(\mu)$ and $\hat{G}(\mu)$, equality in the numbers of men and women who would like to marry *and* can credibly commit to cooperate in marriage is given

[10] Differentiating (20), we get

$$\begin{aligned} 0 &= \{f(V_n) + G(V_c - V_n) + f(z_{nn} - V_n) + \hat{G}(z_{cc} - z_{nn} - (V_c - V_n)]\}dV_n \\ &+\{\int_{-\infty}^{-V_n} g(V_c - \theta)f(\theta)d\theta + \int_{-\infty}^{-U_n} \hat{g}(zcc - V_c + \theta)\hat{f}(\theta)d\theta]\}dV_c \end{aligned}$$

implying that

$$\frac{dV_c}{dV_n} < 0.$$

by a unique value of V_c.

As long as the model is completely symmetric, that is $U^N = V^N$ and, $\forall\theta$, $F(\theta) = \hat{F}(\theta)$, $\forall\mu$, $G(\mu) = \hat{G}(\mu)$, the equilibrium is characterized by equal sharing: $V_c = U_c = z_{cc}/2$ and $U_n = U^N = V_n = V^N = z_{nn}/2$. With these shares, men and women have identical commitment incentives. Hence, the number of men who can credibly commit in marriage equals the number of women who can commit. Such a solution is described by point e in Figure 2.a, where the lines satisfying conditions (19) and (20) intersect. There is a unique symmetric equilibrium. Committed men and women share their marital surplus equally, so that $U_c = V_c = z_{cc}$ / 2. Uncommitted couples play a Nash game in which, due to perfect symmetry, each spouse chooses the same course of action and we get $U_n = U^N = V_n = V^N = z_{nn}$ / 2. Recall that only when the model is completely symmetric does the marriage market equilibrium lie somewhere on the intersection of the two market clearing lines, which occurs within the upper and lower bounds of the return to commitment for men.

Even when we have a pure-sorting equilibrium, there may be an excess supply of men or women in aggregate in the marriage markets because U^N and V^N do not adjust for (20) to hold. In Figure 2.b, I depict a pure-sorting equilibrium in which there is an excess supply of men in total in the marriage market. The equilibrium point e lies on the unique interior value of V_c that ensures (19) holds but it lies above and to the right of points that help the marriage market to clear in the aggregate. Thus, equation (20) does not hold because its LHS exceeds the RHS.

[Figures 2.a and 2.b about here.]

With changes in the distributions of θ and μ, the two lines representing (19) and (20) will shift up or down. For moderate levels of asymmetry, the horizontal line, which ensures the equality of committed men and women among the married, still yields a feasible solution for V_c. Then, the equilibrium still involves pure sorting. The point e' in Figure 3 depicts an equilibrium in which equation (19) still holds within a feasible interior range $[V^N,\ z_{cc} - U^N]$, so that there are equal numbers of committed men and women in the marriage markets but, since the marriage market clearing condition (20) lies above e', there are more women than men in aggregate. Thus, some uncommitted women cannot not marry although they would like to. With greater asymmetry in the model, the equilibrium will become mixed, because (19) will no longer hold in the range $[V^N,\ z_{cc} - U^N]$. That is, when either $F(\theta)$ stochastically dominates $\hat{F}(\theta)$ and $\hat{G}(\mu)$ dominates $G(\mu)$ or $\hat{F}(\theta)$ stochastically dominates $F(\theta)$ and $G(\mu)$ dominates $\hat{G}(\mu)$, there may be a mixed equilibrium where either condition (19.a) or (19.b) will hold. Such a case is illustrated by the point e'' in Figure 4. In this equilibrium, committed men obtain the upper bound on their return to marital commitment, $V_c = z_{cc} - U^N$ and uncommitted men get $V_n = V^N$. The equilibrium point e'' is given by these combinations of unique V_c and V_n. As in Figure 3, (20) does not hold and we have an excess supply of women in the marriage market, as a result of which some uncommitted women cannot not marry.

[Figures 3 and 4 about here.]

Proposition 8 *(1) A pure-sorting equilibrium with equal numbers of total men and women in the marriage market requires complete symmetry so that* $U^N = V^N$ *and* $\forall\ \theta$*„* $F(\theta) = \hat{F}(\theta) \wedge \forall\ \mu$, $G(\mu) = \hat{G}(\mu)$*; it is fully characterized by (8), (9), (19) and (20); and it generates* $V_c = U_c = z_{cc}/2$ *and* $V_n = U_n = z_{nn}/2$*;*

(2) A mixed equilibrium with a surplus of committed men in the marriage market requires $F(\theta)$ *to stochastically dominate* $\hat{F}(\theta)$ *and/or* $\hat{G}(\mu)$ *to dominate* $G(\mu)$ *such that (19.a) holds with* $V_c = V^N$ *and* $U_c = z_{cc} - V^N$*; this equilibrium is fully characterized by (8), (9), (10) and (19.a);*

(3) A mixed equilibrium with a shortage of committed men in the marriage market requires $\hat{F}(\theta)$ *to stochastically dominate* $F(\theta)$ *and/or* $G(\mu)$ *to dominate* $\hat{G}(\mu)$ *such that (19.b) holds with* $U_c = U^N$ *and* $V_c = z_{cc} - U^N$*; it is fully characterized by (8), (9), (11) and (19.b).*

Proof. *See Appendix Section 5.2.* ■

In sum, when there is an excess supply of committed women in the marriage market, the return to commitment for all men rises. This creates higher incentives for men to commit to ex-ante marital allocations determined in the marriage market despite the fact that the distributions for marriage preference and costs of commitment yield higher costs and lower marriage preference among the men. Also, some men or women who would like to marry but not commit in marriage may not find a partner because, while the prices of committed men and women (i.e., U_c and V_c) adjust with imbalances in demand and supply, those of uncommitted spouses (i.e., U^N and V^N)

do not. As a result, regardless of whether the equilibrium is mixed or pure, there can be an excess supply of men or women in aggregate in the marriage market.

2.6 Impact of the Sex Ratio

Although I assumed so far that there are equal numbers of men and women in the population, one can easily extend the analysis to examine the impact of an uneven sex ratio. Let $r \gtreqless 1$ represent the ratio of men to women in the population. Then, we modify equations (19) and (20) as follows, respectively:

$$r \int_{-\infty}^{\infty} G(V_c + \theta) f(\theta) d\theta \; = \; \int_{-\infty}^{\infty} \hat{G}(U_c + \theta) \hat{f}(\theta) d\theta, \tag{19'}$$

$$r \left[F(V^N) \; + \; \int_{-\infty}^{-V^N} G(V_c + \theta) f(\theta) d\theta \right] \; = \; \hat{F}(U^N) \; + \; \int_{-\infty}^{-U^N} \hat{G}(U_c + \theta) \hat{f}(\theta) d\theta \; . \tag{20'}$$

Note that, even if $U^N = V^N$ and, $\forall\ \theta,\ F(\theta) = \hat{F}(\theta),\ \forall\ \mu,\ G(\mu) = \hat{G}(\mu)$, the equilibrium with an uneven sex ratio will not be characterized by equal sharing. For example, if $r > 1$ and there are more men than women in the population, then (20') implies that V_c will need to decline and U_c will need to rise to ensure that there are equal numbers of men and women who can commit. As a result, the marriage-market return for commitment of the sex in excess supply will fall and that of the sex in short supply will rise, regardless of whether the marriage market equilibrium is strict or mixed.

For r closer to unity, equation (19') may still continue to hold and a strict sorting equilibrium with equal numbers of educated men and educated women among the married emerging in equilibrium. However, with more uneven sex ratios, equation (19') may not hold even if $U^N = V^N$ and, $\forall\ \theta,\ F(\theta) = \hat{F}(\theta),\ \forall\ \mu,\ G(\mu) = \hat{G}(\mu)$. Then, there will be a mixed equilibrium in which some committed men marry uncommitted women. Whether there will be equal numbers of men and women in total in the marriage market will be independent of market clearing among the committed spouses and (20') may fail to hold even in a pure-sorting equilibrium. However, ceteris paribus, increases (decreases) in r would tend to create an excess supply of both committed men (women) and uncommitted men (women) in the marriage market. Consequently, increases in the sex ratio r would make it more likely that the RHS of both equations (19') and (20') exceed the LHS.

In Figure 5, I depict the impact of a rise in the sex ratio on the marriage market equilibrium. The figure is drawn under the assumption that the market is completely symmetric initially; $U^N = V^N$ and, $\forall\ \theta$, $F(\theta) = \hat{F}(\theta)$, $\forall\ \mu$, $G(\mu) = \hat{G}(\mu)$. Thus, when the sex ratio r equals one, we have the pure-sorting equilibrium with marriage market clearing in the aggregate depicted in Figure 2.a; that is, there are equal numbers of men and women in the marriage market among whom there are also equal numbers of men and women willing to commit to the ex-ante marriage prices. When the sex ratio r rises above one, then (19') implies that V_c will need to drop and U_c will need to rise to compensate committed women for the increase in the number of men who are willing to commit to cooperative behavior. As a result, the horizontal line which represents market clearing among the committed individuals will need to shift lower. By the same token, since V_c and V_n are substitutes in determining men's marriage incentives, V_n should decrease (holding constant V_c) or V_c should decrease (holding constant V_n) for the total numbers of men and women in the marriage market to equal one another. This indicates the aggregate marriage market clearing line should shift downward and to the left. However, given that V_n and U_n are determined by non-cooperative Nash behavior, they will not adjust with changes in r. Thus, according to (20'), there will now be an excess supply of men in the marriage markets. For moderate changes in r, the equilibrium will still yield pure sorting so that (19') still attains. As shown in Figure 5, there will be equal numbers of committed men and women in the marriage market, but due to higher r, the former will get a smaller share of the marital surplus. In aggregate, there will be a higher number of men who would like to marry but cannot commit than the number of women who would like to do the same.

[Figure 5 about here.]

Lemma 9 *With an uneven sex ratio, $r \neq 1$, a mixed equilibrium can be attained even if $U^N = V^N$ and $\forall\ \theta,\ \mu$, $F(\theta) = \hat{F}(\theta)$, $G(\mu) = \hat{G}(\mu)$.*

Proof. *Follows immediately from Proposition 8 and equations (19') and (20').* ∎

3 An Example

Consider the following simple model to trace the main implications of the model: Assume that, irrespective of the differences in labor market wages or household roles, men and women have the same preferences given by

$$u = v = \begin{cases} cq + \theta & \text{if individual } i \text{ is not committed,} \\ \\ cq + \theta - \mu & \text{if individual } i \text{ is committed,} \end{cases} \tag{21}$$

where c is a private good, q is a public good that can be shared if two people marry but is private if they remain single.

The household public good is produced according to a production function

$$q = t \ , \tag{22}$$

where t represents time spent in home production. This specification reflects transferable utility between spouses.

All individuals are endowed with one unit of time. The wage rate for all men is identical and equal to w^m and that for all women is identical and equal to w^w with $w^m > w^w$.

3.1 Cooperation and Commitment

If two committed spouses marry each other in a *pure-sorting equilibrium*, they maximize

$$\max_{q, c_i + c_j} q(c_i + c_j) + \theta_i + \theta_j - \mu_i - \mu_j \tag{23}$$

subject to (22) and their budget constraint

$$c_i + c_j \leq w^m(1 - t_i) + w^w(1 - t_j) \ . \tag{24}$$

The efficient household division of labor then involves the husband specializing in market work ($t_i = 0$) and the wife undertaking home production ($t_j = 1$) so that the maximized material output is

$$(c_i + c_j)q = w^m \equiv \zeta_{cc} \ . \tag{25}$$

In contrast, a single man i solves

$$\underset{q_i, c_i}{Max} \;\; c_i q_i \tag{26}$$

subject to $q_i = t_i$ and

$$c_i = w^m(1 - t_i). \tag{27}$$

His optimal behavior generates a utility level of $\zeta_m = w^m/4$. A single woman j solves an analogous problem and obtains $\zeta_w = w^w/4$. Therefore, the total *marital surplus* generated by a cooperative marriage mode is

$$s_{ij} \equiv \; z_{cc} + \theta_i + \theta_j \; - \mu_i - \mu_j \; = \frac{3w^m - w^w}{4} + \theta_i + \theta_j \; - \mu_i - \mu_j \; , \tag{28}$$

Due to the fact that the equilibrium involves pure sorting, we get $U_c = V_c = z_{cc}/2$ $= (3w^m - w^w) \; / \; 8$.

3.2 Nash-bargaining

Next consider a couple who cannot commit to efficient household roles ex ante in a *pure-sorting equilibrium*. Each spouse in such a marriage recognizes that his or her partner is not willing to abide by the ex-ante agreement regarding spousal roles and allocations. Instead, each partner maximizes his or her own utility given by (21) subject to (22), (27) and taking as given his/her spouses' choices. That is, an uncommitted man i married to an uncommitted woman j solves

$$\underset{q, c_i}{Max} \;\; c_i q + \theta_i \tag{29}$$

subject to equation (27), and

$$q = t_i + \bar{t}_j, \tag{30}$$

The wife j solves an analogous problem and the optimal time allocation of the couple $(i,\, j)$ yields $t_i = t_j = 1/3$. Consequently, $U_n = U^N = 7w^w/36$, $\zeta_m + U_n =$ $4w^w/9$, $V_n = V^N = 7w^m/36$, and $\zeta_w + V_n = 4w^m/9$. Hence, the maximized material output of the couple is

$$(c_i + c_j)q = \frac{4(w^m + w^w)}{9} \equiv \; \zeta_{nn}. \tag{31}$$

The total *marital surplus* generated by their marriage is then given by

$$s_{ij} = \frac{7(w^m + w^w)}{36} + \theta_i + \theta_j \; \equiv \; z_{nn} + \theta_i + \theta_j \; . \tag{32}$$

3.3 Opportunistic Mode

In *mixed-marriage equilibria*, either committed men marry uncommitted women or uncommitted men marry committed women. Consider first the marriage of a committed woman j and an uncommitted man i, and the scenario in which j abides by her commitment as in Definition 3. Since $w^m > w^w$, woman j would supply all of the home production time ($t_j = 1$) but man i, given that he is not willing to commit, would maximize his private consumption taking as given his wife's full-time commitment to home production. That is, man i would solve

$$\underset{t_i,c_i}{Max} \;\; c_i q + \theta_i \tag{33}$$

subject to

$$c_i \; + c_j \;\; = \;\; (1 - t_i) w^m \;, \tag{34}$$

$$q = t_i + 1 \;, \tag{35}$$

and

$$\zeta_w = \frac{w^w}{4} \quad \wedge \quad U_c \;=\; 0 \quad \Rightarrow \quad c_j \;\;=\;\; \frac{w^w}{4(t_i+1)} \;. \tag{36}$$

As a result, we get $t_i = 0$, $c_j = w^w/4$ and $c_i = w^m - w^w/4$, which yield $V_n = z_{nc}$, and $\zeta_m + V_n = \zeta_m + z_{nc}$, with the total *marital surplus* generated by the marriage now being equal to

$$s_{ij} = \frac{3w^m - w^w}{4} + \theta_i + \theta_j \; - \mu_j \;\equiv\; z_{nc} + \theta_i + \theta_j \; - \mu_j \;. \tag{37}$$

Similarly, when a committed man i and an uncommitted woman j get married in a mixed equilibrium and man i is willing to remain committed, woman j chooses t_j in order to maximize her own utility taking as given $\bar{t}_i = 0$:

$$\underset{q,c_j}{Max} \;\; c_j q + \theta_j \tag{38}$$

subject to

$$c_i + c_j \leq w^m \;, \tag{39}$$

$$q = t_j \ , \tag{40}$$

and

$$\zeta_m = \frac{w^m}{4} \quad \wedge \quad V_c \ = \ 0 \quad \Rightarrow \quad c_i \ = \ \frac{w^m}{4t_j} \ . \tag{41}$$

This problem generates $t_j = 1$, $c_i = w^m/4$, and $c_j = 3w^m/4$. As a result, we have, $\zeta_m + V_c = \zeta_m = \gamma w^m/4$ and $\zeta_w + U_n = \zeta_w + z_{cn}$. The total *marital surplus* now equals

$$s_{ij} = \frac{3w^m - w^w}{4} + \theta_i + \theta_j \ - \mu_j \ \equiv \ z_{cn} + \theta_i + \theta_j \ - \mu_i \ . \tag{42}$$

Note that $z_{cc} = z_{nc} = z_{cn} > z_{nn}$ but in both kinds of marriage either $U_c = 0$ or $V_c = 0$. Thus, neither of the two outcomes defined above are sustainable and both couples will revert to Nash Bargaining defined by (29) through (32).[11]

3.4 Equilibrium

Next, consider uniform distributions of θ and μ such that they respectively have $[-a, a]$ and $[-b, b]$ as their supports of the lower and upper bounds, where $b \geqslant w^m$, a. Under strictly positive assortative mating, the numbers of men and women who are willing to marry and commit are equal. Then, according to (19)

$$\frac{(V_c + a + b)^2 - (V_c - a + b)^2}{8ab} \ = \ \frac{(U_c + a + b)^2 - (U_c - a + b)^2}{8ab}. \tag{19.c}$$

Equation (19.c) simplifies to $(V_c + b) \ / \ 2b = (U_c + b) \ / \ 2b$ and, given the identical distributions of θ and μ, it is clear that (19.c) will hold if and only if $V_c = U_c$.

[11] From equations (28) and (32) we get

$$z_{cc} + z_{nn} = \frac{3w^m - w^w}{4} + \frac{7(w^m + w^w)}{36}$$

and

$$z_{cn} + z_{nc} = \ 2z_{nn} = \frac{7(w^m + w^w)}{18} \ .$$

Hence, $z_{cc} + z_{nn} > z_{cn} + z_{nc}$ and positive sorting equilibria holds.

For the numbers of men and women who are on the marriage market to be equal, we need

$$\frac{V^N + a}{2a} + \frac{(V_c - V^N + b)^2 - (V_c - a + b)^2}{8ab} = \frac{U^N + a}{2a} + \frac{(U_c - U^N + b)^2 - (U_c - a + b)^2}{8ab} . \tag{20.c}$$

However, given that men's wage exceed women's, we have $V_n > U_n$ for uncommitted couples who play a non-cooperative Nash game between them. Consequently, (20.c) cannot hold because $V_c = U_c = z_{cc}/2 = (3w^m - w^w] \ / \ 8$ and $V_n = V^N = 7w^m/36 > U_n = U^N = 7w^w/36$. Thus, the LHS of (20.c) exceeds the RHS and there are more men than women in the marriage markets. This yields a pure sorting equilibrium in which a fraction of all marriages involve cooperative behavior, the rest are characterized by non-cooperative Nash outcomes, and there are excess men in the marriage markets, as depicted in Figure 3. As (19.c) indicates, the higher are the gains from marriage, the wider is the distribution of θs, and the narrower that of μ, the higher is the fraction of all marriages in which spouses commit and behave cooperatively.

With $V_c = U_c = z_{cc}/2 = (34w^m - w^w) \ / \ 8$, the proportion of committed men and women in the marriage market equals

$$\frac{1}{2} + \frac{3w^m - w^w}{16b} < 1 , \tag{17.a}$$

and the fraction of all men and women (both committed and uncommitted) in marriage market respectively equals

$$\frac{324a(3w^m - w^w + 24b) + 7(9w^m - 20w^w + 72b)w^m - 1296a^2}{10368ab} , \tag{18.a}$$

and

$$\frac{324a(3w^m - w^w + 24b) + 7(16w^w - 27w^m + 72b]w^w - 1296a^2}{10368ab} . \tag{18.b}$$

Equations (18.a) and (18.b) yield equal aggregate numbers of men and women in the marriage markets only if the model is completely symmetric so that, in addition to θ and μ having gender neutral distributions, we have $w^m = w^w$. But with $w^m \neq w^w$, the total numbers of men and women in the market will differ even though there are equal numbers of committed men and women available for marriage. And if $w^m > w^w$ as I have assumed in this example, then (18.a) will exceed (18.b) and there will be more men than women in the marriage market. In sum, this example characterizes an pure-sorting equilibrium in which there are, in aggregate, more men than women in the marriage market.

For heuristic purposes, now take the case in which the distributions of μ differ by gender so that for men we have $[-b,\, b+\delta]$, $0 < \delta$, and for women we still have $[-b,\, b]$, as the supports of the lower and upper bounds of μ. Under strictly positive assortative mating, the numbers of men and women who are willing to marry and commit are equal. Then, (19) generates

$$\frac{(V_c + a + b)^2 - (V_c - a + b)^2}{8ab + 4a\delta} = \frac{(U_c + a + b)^2 - (U_c - a + b)^2}{4ab}. \qquad (19.d)$$

For the numbers of men and women who are on the marriage market to be equal, we need

$$\frac{V^N + a}{2a} + \frac{(V_c - V^N + b)^2 - (V_c - a + b)^2}{8ab + 4a\delta} = \frac{U^N + a}{2a} + \frac{(U_c - U^N + b)^2 - (U_c - a + b)^2}{8ab}. \qquad (20.d)$$

For δ close to zero, the analysis above would still hold and a pure sorting equilibrium with an excess supply of men over women will continue to be sustained (due to the fact that $w^m > w^w$). With higher δ, men become more reluctant to commit. As a consequence, V_c will rise and U_c will fall in order to keep (20.d) satisfied. However, with sufficiently large δ, (20.d) will no longer be satisfied evaluated at $V_c = z_{cc} - U^N$ and $U_c = U^N$. Hence, we find that if men's distribution of commitment costs significantly dominate that for women in a first-order stochastic sense, then there will be more committed women than men but also more total men than women in the marriage markets. Consequently, some men who would like to marry but cannot commit will remain single in equilibrium. In sum, this defines a mixed equilibrium where there is an excess supply of women in the marriage market among whom there are more of them willing to commit than men are willing to do so.

4 Conclusion

The traditional approach to analyze household choices takes the family as the relevant decision-making unit, but since the early-1990s, the non-unitary household models have emerged as the compelling theoretical alternative for analyzing the economics of the family. The existing literature provides three distinct but potentially valid classes of non-unitary models: First, there are the 'collective models' where household members act collectively to choose from a host of Pareto efficient choices. Second, we have the 'bargaining models' in which the household is an institution in which a cooperative bargaining process determines allocations. Finally, there are models of non-cooperative Nash bargaining where household members choose their actions taking as given those of the others in the family. Typically, while the first two types of non-unitary models yield efficient outcomes, those that belong in the third class do not.

The collective models depend on the Pareto optimality of household decisions and their testable predictions and identification power rest on whether or not household members can cooperate to generate those efficient outcomes. Hence, their main drawback stems from the fact that household choices need to be confined to only those on the Pareto frontier. When spousal choices such as labor supply and production specialization could influence not only household income but allocations within it, cooperative behavior would be harder to sustain because it could be costly for household members to commit to efficient choices ex ante. Then, one can no longer restrict attention to the efficient frontier in which case a plausible method to deal with this complication is to model spousal behavior as a non-cooperative bargaining process. In general, there is little guidance about which of these two modes of household behavior is more relevant and appropriate.

In this paper, I propose an intra-household allocation model in which spousal matching is endogenous and the cost of commitment to the Pareto efficient household roles and intra-marital allocations varies across all individuals. Such a model helps one to uncover the underlying determinants of cooperative household behavior. The benefit of extending the family economics literature in this fashion is due to the fact that there exists no current consensus on whether the family should be modeled as a cooperative institution or one in which partners bargain over resource allocation without a commitment to cooperate.

The main findings of the paper can be summarized as follows: When marriage preferences and commitment costs are gender symmetric, there is a pure sorting equilibrium in which some couples do not cooperate and some do. In such an equilibrium, the marital gains are shared equally between the husbands and the wives, although the surplus generated by cooperative marriages is higher. When marriage preferences or commitment costs are not gender neutral, however, there are also mixed marriages in which spouses of the sex with lower costs and greater willingness to marry will choose to cooperate while their spouses will not. In such marriages, the partners with the higher costs or lower eagerness to marry will capture all of the marital gains. Due to spousal competition in the marriage markets, committed spouses of the same sex will appropriate all of their marital surplus as well. In mixed equilibria, the incentives for commitment will be at a maximum also because, in the marriage markets, there will be an excess supply of individuals of the sex with fewer committed individuals. If the gains from marriage fall, then not only will fewer individuals marry, but also more spouses will choose to enter marriages without a commitment. An excess supply of men in a marriage cohort reduces the commitment incentives of men and raises those of women. The corollary of this finding is that, if there are more men than women in a given marriage cohort, then the number of men in the marriage market will exceed women in the market, but not as much as the level implied by the sex ratio. The reason is that the incentives to marry drops for those on the long side of the market, regardless of whether they act cooperatively or opportunistically in their marriages.

In sum, the main contribution of the model I presented is to identify the circumstances under which a given couple is more likely to adopt cooperative behavior as opposed to non-cooperative Nash play in deciding household allocations. The factors that influence the household mode of behavior range from gender asymmetries in the costs of commitment and the propensity to marry, to changes in the home production technology and the sex ratio in the marriage markets. An important insight provided by the model is that, while the price of committed spouses is determined in the marriage markets and that of non-committed spouses is not, the propensity to not commit to ex-ante expected allocations entering a marriage and act opportunistically during it are influenced by the changes in the price of commitment.

5 Appendix

5.1 Proving that $V_c \geqslant V_n$ and $U_c \geqslant U_n$:

- If there is a mixed equilibrium with more committed men than committed women, we have $V_c = V_n = V^N$, $U_n = U^N$, and $U_c = z_{cc} - V^N$. Since $z_{cc} > z_{nn}$, $U_n + V_n = U^N + V^N = z_{nn} \Rightarrow U_n = z_{nn} - V^N$, it follows that $U_c = z_{cc} - V^N > U_n$. Hence, $V_c = V_n$ and $U_c > U_n$.

- If there is a mixed equilibrium with more committed women than committed men, $U_c = U_n = U^N$, $V_n = V^N$, and $V_c = z_{cc} - U^N$. Since $z_{cc} > z_{nn}$, $U_n + V_n = U^N + V^N = z_{nn} \Rightarrow V_n = z_{nn} - U^N$, it follows that $V_c = z_{cc} - U^N > V_n$. Hence, $U_c = U_n$ and $V_c > V_n$.

- If there is a pure-sorting equilibrium, then only (9) and (10) hold so that $U_n + V_n = U^N + V^N = z_{nn}$ and $U_c + V_c = z_{cc}$. Given (2), $z_{cc} > z_{nn}$. Moreover, stability in the marriage markets, together with spousal commitment to efficient outcomes, suggests no committed spouse can be worse off materially than he or she could be in another marriage. Thus, either $V_c \geqslant V_n$ and $U_c > U_n$, or $V_c > V_n$ and $U_c \geqslant U_n$, or $V_c > V_n$ and $U_c > U_n$ ■

5.2 Proof of Proposition 8:

- (1) If $U^N = V^N$ and, $\forall\ \theta$, $F(\theta) = \hat{F}(\theta) \wedge \forall\ \mu$, $G(\mu) = \hat{G}(\mu)$, then (19) holds with $V_c = U_c = z_{cc}/2$ and (20) holds with $V_c = U_c = z_{cc}/2$ and $V_n = U_n = z_{nn}/2$. As a result, there are equal numbers of *committed* men and committed women as well as equals numbers of *total* men and women in the marriage market. Consequently, equations (8), (9), (19) and (20) define a pure-sorting equilibrium in which $V_c = U_c = z_{cc}/2$ and $V_n = U_n = z_{nn}/2$;

- (2) If $F(\theta)$ to stochastically dominates $\hat{F}(\theta)$ and/or $\hat{G}(\mu)$ to dominates $G(\mu)$, then (19) evaluated at $V_c = U_c = z_{cc}/2$ generates

$$\int_{-\infty}^{\infty} G\left(\frac{z_{cc}}{2} + \theta\right) f(\theta) d\theta \ > \int_{-\infty}^{\infty} \hat{G}\left(\frac{z_{cc}}{2} + \theta\right) \hat{f}(\theta) d\theta. \tag{A.1}$$

Given $F(\theta)$, $\hat{F}(\theta)$, $\hat{G}(\mu)$ and $G(\mu)$, if $\nexists\ V_c \in (V^N, z_{cc}/2] \wedge U_c \in [z_{cc}/2, z_{cc} - V^N)$, then (19) cannot hold in equilibrium but (19.a) will. If (19.a) holds, then $V_c =$

V^N and $U_c = z_{cc} - V^N$. As a result, (8), (9) and (10), together with (19.a), fully characterize this mixed equilibrium, in which there are more committed men than committed women.

- (3) If $\hat{F}(\theta)$ to stochastically dominates $F(\theta)$ and/or $G(\mu)$ to dominates $\hat{G}(\mu)$, then (19) evaluated at $V_c = U_c = z_{cc}/2$ generates

$$\int_{-\infty}^{\infty} G\left(\frac{z_{cc}}{2} + \theta\right) f(\theta)d\theta \; < \; \int_{-\infty}^{\infty} \hat{G}\left(\frac{z_{cc}}{2} + \theta\right) \hat{f}(\theta)d\theta. \tag{A.2}$$

Given $F(\theta)$, $\hat{F}(\theta)$, $\hat{G}(\mu)$ and $G(\mu)$, if $\nexists\, U_c \in (U^N,\, z_{cc}/2] \wedge V_c \in [z_{cc}/2,\, z_{cc} - U^N)$, then (19) cannot hold in equilibrium but (19.b) will. If (19.b) holds, then $U_c = U^N$ and $V_c = z_{cc} - U^N$. As a result, (8), (9) and (10), together with (19.b), fully characterize this equilibrium with more committed women than committed men.

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Figure 1: Marriage and Commitment Decisions of Men

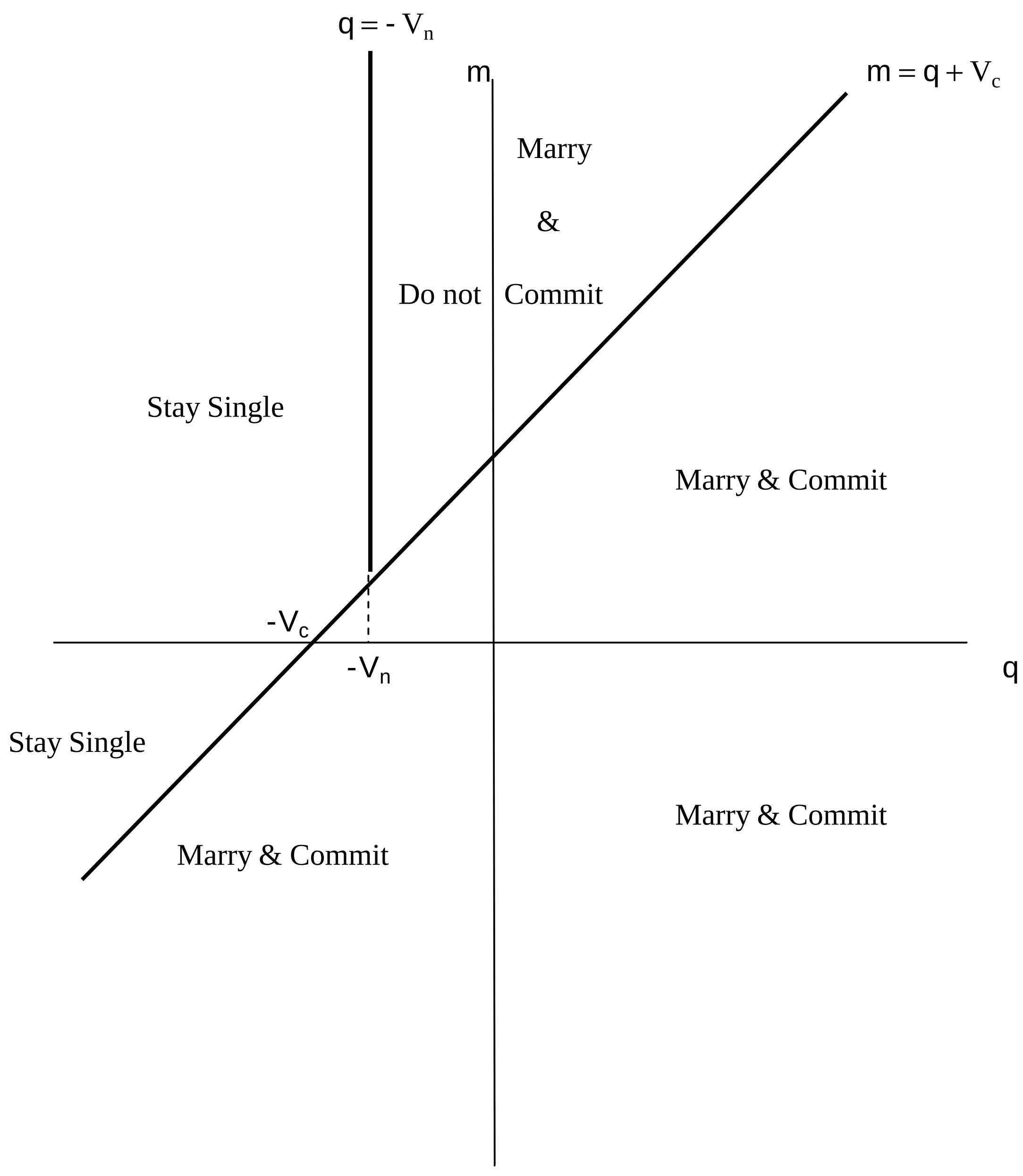

Figure 2.a: A Pure-Sorting Equilibrium with Aggregate Market Clearing

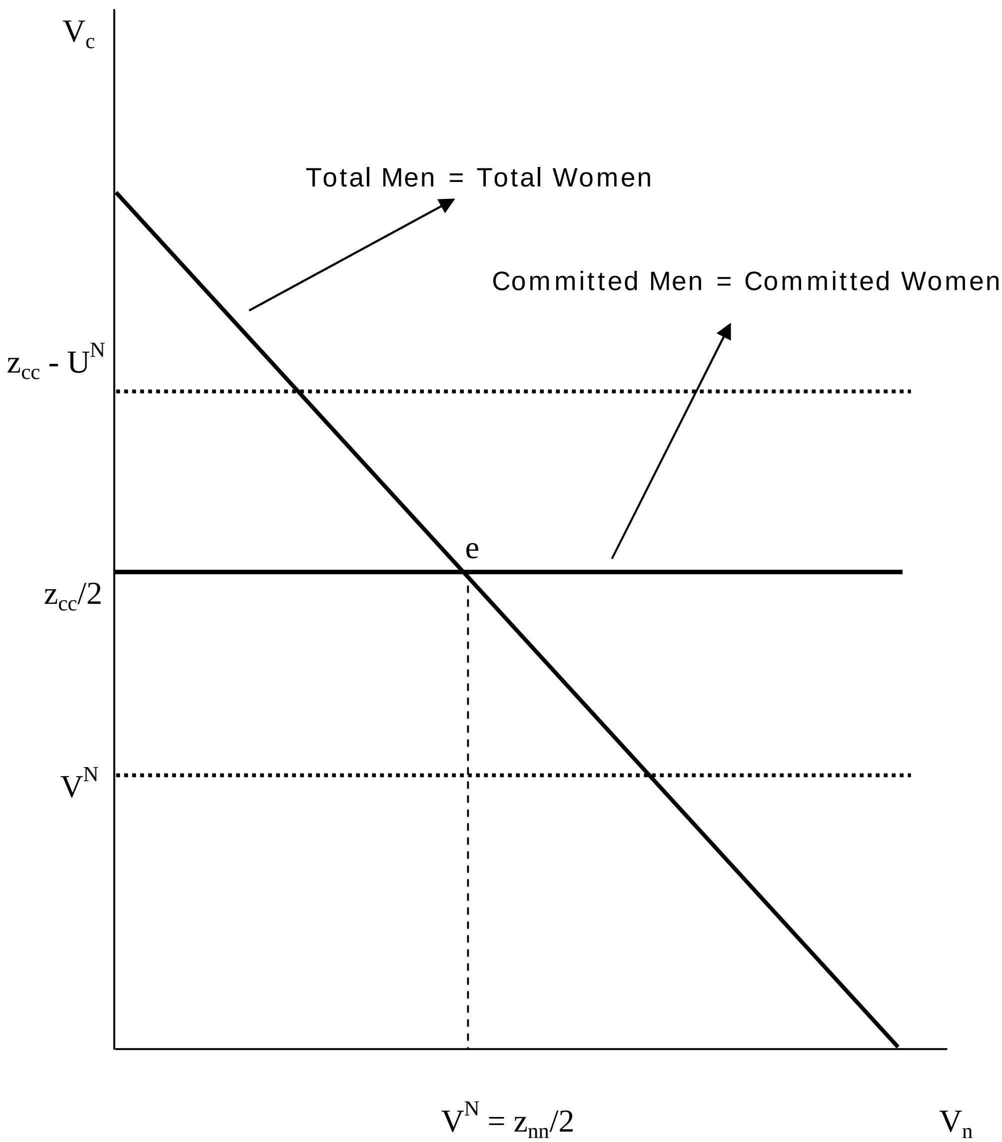

Figure 2.b: A Pure-Sorting Equilibrium with an Excess Supply of Men

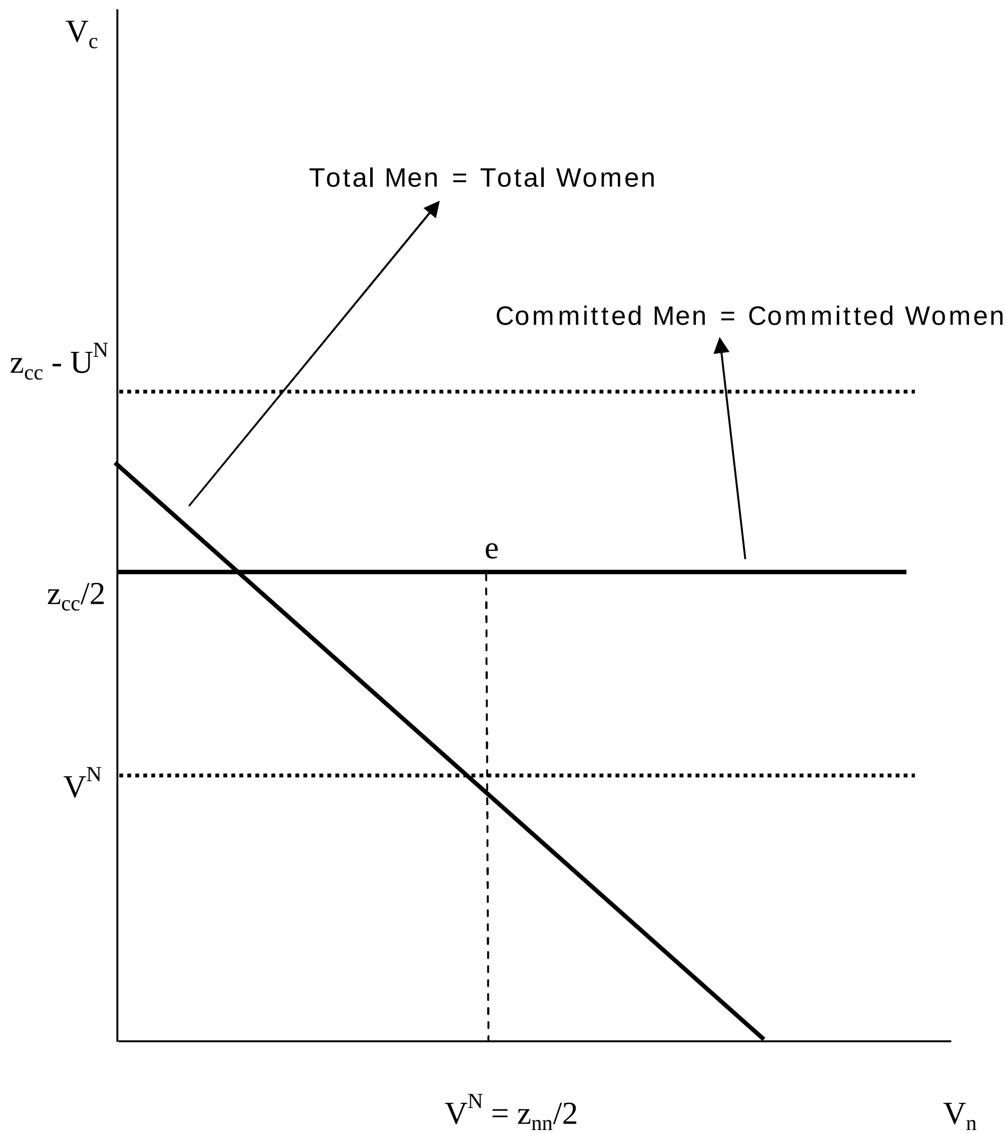

Figure 3: An Asymmetric Pure-Sorting Equilibrium with an Excess Supply of Women

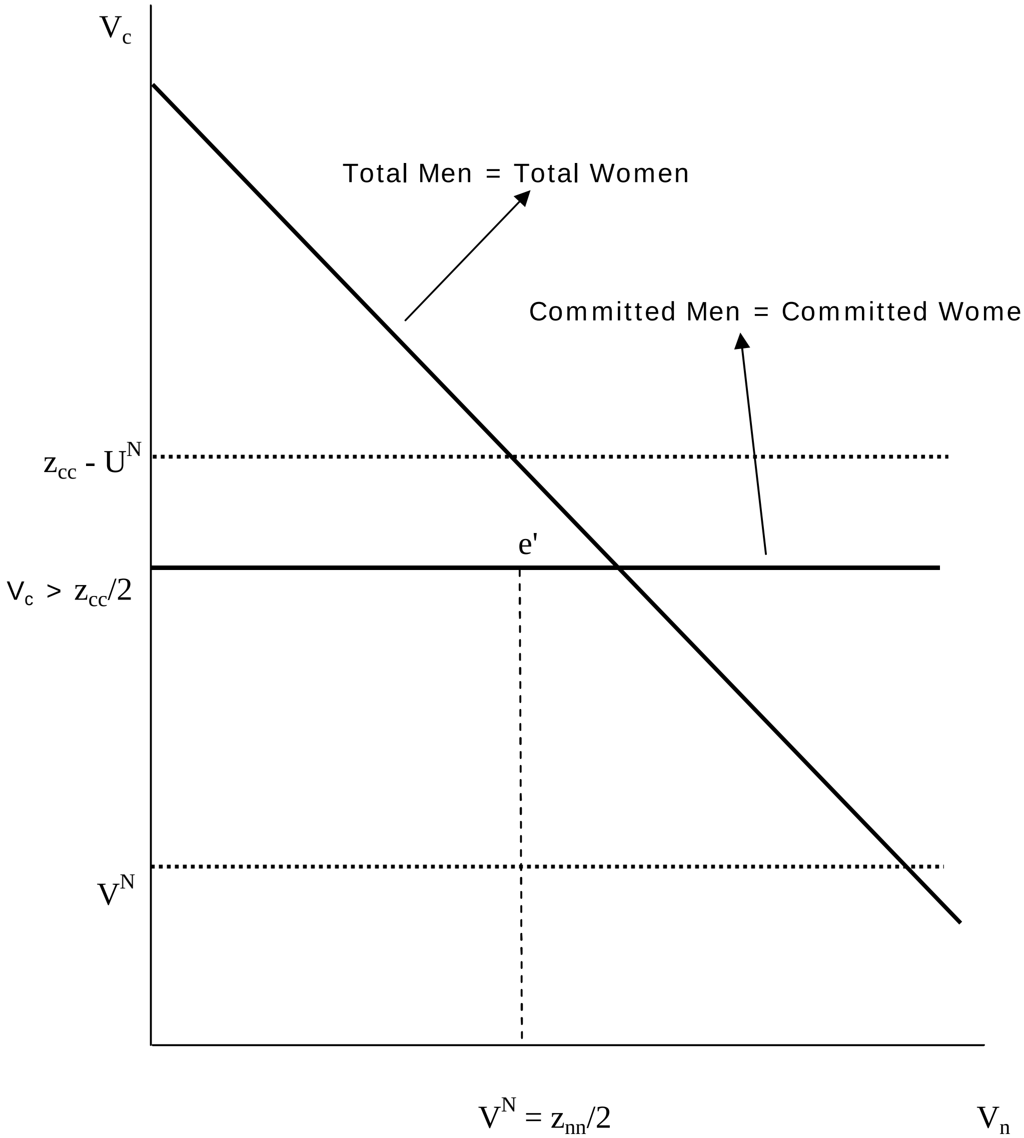

Figure 4: A Mixed Equilibrium with an Excess Supply of Women

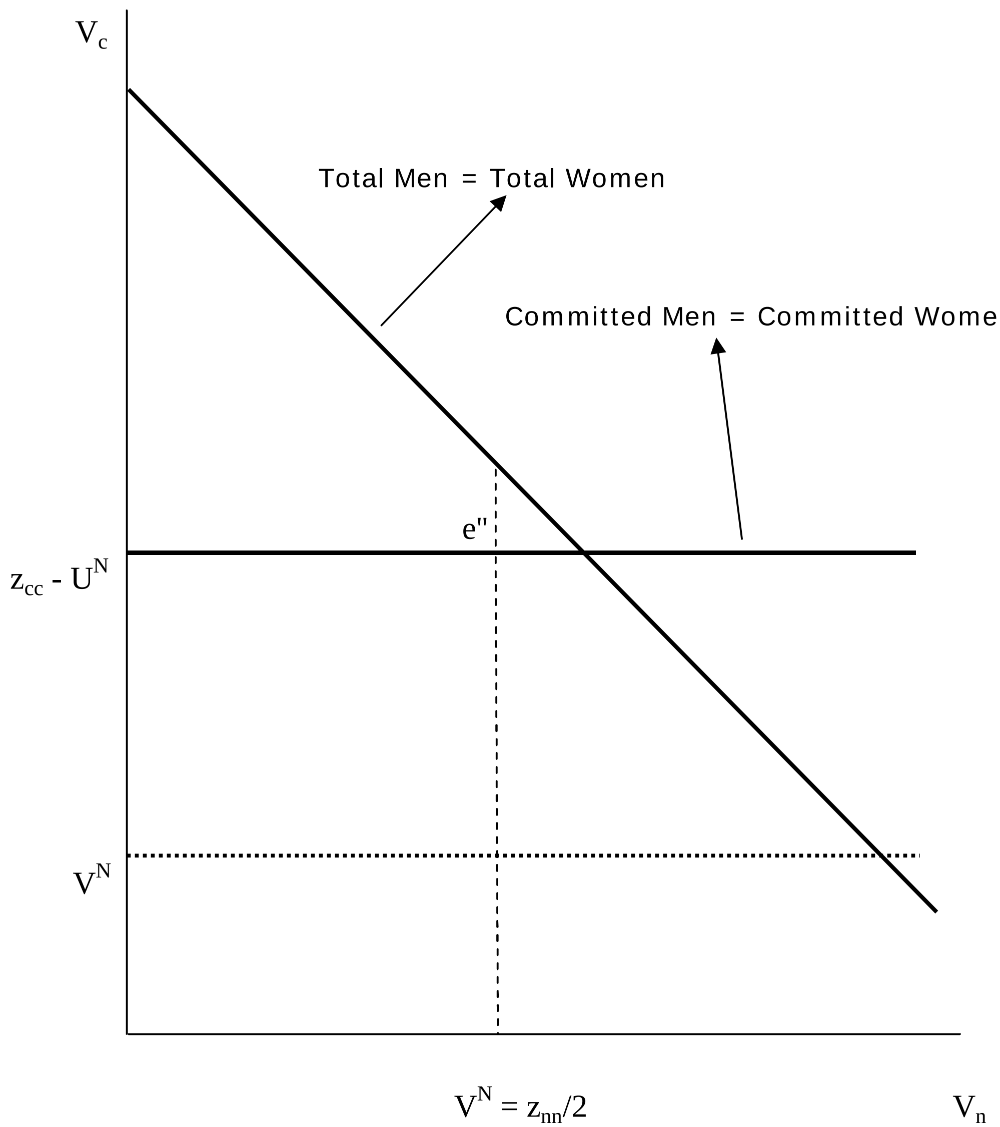

Figure 5: The Impact of an Increase in the Sex Ratio r