Paying Customers to Switch

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This paper studies the business practice of offering discounts to new customers in markets with switching costs. In a two-period homogeneous-good duopoly model, it is shown that the equilibrium amount of discounts increases continuously in the expected switching costs of a typical consumer. In equilibrium, firms offer the same prices and discounts in a mature market even if they have different market shares, and the demands faced by these firms in a new market become more elastic. Firms are worse off engaging in the discriminatory pricing, while consumers need not necessarily benefit from it. There is costly equilibrium switching of consumers, which creates a dead-weight loss to the society.

1. Introduction

In markets with repeated purchases, consumers often need to incur switching costs when changing from one supplier to another, such as the costs of transactions or the costs of learning to use a new product. The effects of such consumer switching costs on the equilibrium outcomes of these markets have received wide attention in the literature recently.\(^1\) The analysis in this literature, however, typically assumes that a firm has to charge the same prices for all its customers. While this is clearly an important case to be studied, and perhaps is an useful benchmark to start with, there are also many interesting situations where firms are able to charge different prices to existing and new customers. In fact, it seems

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rather natural that firms will be able to use such pricing policies in markets with consumer switching costs, since the presence of switching costs naturally separates a firm's potential (perhaps otherwise identical) customers into two different groups. A familiar example of this is the market for long-distance telephone service in the United States, where competing firms routinely provide discounts (monetary payments) to new customers for switching from a competitor(s), and thus effectively charge lower prices to these customers.

In this paper, I shall call the practice of offering discounts to new customers in markets with repeated purchases "paying customers to switch," or PCTS for short. In standard economic terms, it is obviously a form of third-degree price discrimination. The popularity of such pricing practices raises several economic questions. First, how are prices determined under PCTS, and how do they differ from those under a pricing regime where all customers have to be treated equally (which I shall call UNIF in this paper)? Of particular interest is the issue of how a firm's market share affects its pricing behavior. The answer to this is interesting not only because it sheds light on price competition in a mature market where each firm has established a market share, but also because it can have implications for consumer demand and the price competition in a new market. Second, what is the relationship between the use of PCTS and the presence of consumer switching costs? Or, to pose the question in another way, can switching costs explain the wide use of PCTS in markets with repeated purchases? Third, how does the practice of PCTS affect firms and consumers? Since a firm can always choose to provide zero discount to new customers, it is tempting to conjecture that firms will have higher profits if they are able to separate new and old customers and then choose to offer new customers discounts. While such reasoning seems perfectly valid for a monopoly firm, it could be misleading in an oligopoly market, as is shown in Homes (1989), and a model will thus be needed before we can reach a conclusion. Similarly, it is also not obvious how consumers will be affected. Finally, from the perspective of public policy, it would be desirable to assess the effects of such business practices on social welfare.

I address these issues in a simple two-period duopoly model, which is spelled out as follows: The two firms, named A and B, produce a homogeneous product with a constant marginal cost, \( c \geq 0 \). There is a continuum of consumers of measure 1 in the market, each of whom has an unit demand per period for the product with a reservation value \( R \), where \( R > c + \theta \) and \( \theta > 0 \). In the first period, both firms compete in prices, resulting in a proportion \( \alpha \) of the consumer population buying from A and the rest of the consumer population, \( 1 - \alpha \), buying from B, where \( 1 \geq \alpha \geq 0 \). I shall call \( \alpha \) firm A's market share and \( 1 - \alpha \) firm
B's market share at the beginning of the second period. The identity of a consumer regarding from which firm she bought in the first period is assumed to be known to both firms. In the second period, if a consumer switches from the firm she bought from previously to buying from another firm, she incurs a switching cost, $s$, which she learns privately at the beginning of the second period. For convenience, each consumer's $s$ is assumed to be an independent realization of a random variable $S$ that is uniformly distributed on $[0, \theta]$ across the consumer population. Firms again compete in prices in the second period, but under the $pcts$ regime, a firm can provide a monetary payment to a customer who switches away from its rival, while under the $unif$ regime such payment is not allowed. Both firms and consumers discount their second-period payoffs with the discount factor $\delta \in (0, 1]$.

In suggesting the model above, great emphasis has been placed on the tractability of the analysis. My aim here is to have a model that can capture some important aspects of markets with repeated consumer purchases and is yet simple enough to permit closed-form solutions. Explicit comparisons of the outcomes under $pcts$ with those under $unif$ would then become possible. Some of the modeling issues will be discussed later, in Section 4.

The analysis for the mature market (the second period) is carried out in Section 2. A central finding there is that the equilibrium prices of both firms under $pcts$ are independent of their respective market shares, in sharp contrast to the result under $unif$, where a firm with a higher market share charges higher prices. Both firms' prices are higher than the marginal cost but below the monopoly price. Both prices and new-customer discounts are increasing functions of switching costs, and as the switching costs approach zero ($\theta \to 0$), the equilibrium prices and new-customer discounts approach continuously the marginal cost and zero respectively. The profits of each firm are always lower under $pcts$ than under $unif$ in the mature market. Although in equilibrium both firms offer the same prices and discounts under $pcts$, there are more consumers who switch suppliers under $pcts$ than under $unif$, resulting in a higher deadweight loss to the society.

Section 3 studies the new market (the first period), where no consumer is attached to any firm in any way. A point of importance is that consumer demand in the new market is not affected by the presence of switching costs in the mature market under $pcts$, while under $unif$ consumer demand in the new market becomes less elastic. A unique subgame-perfect equilibrium exists under $pcts$, while there are multiple subgame perfect equilibria under $unif$. Firms can also have lower prices and profits in the new market under $pcts$, and the discounted sum of
each firm's profits is lower under PCTS. However, consumers may or may not be better off under PCTS, and there is a higher dead-weight loss to the society due to more equilibrium switching of consumers under PCTS.

Section 4 discusses the robustness of some results, the assumptions of the model, and possible directions for future research.

The analysis here is closely related to previous studies of markets with endogenous switching costs. Banerjee and Summers (1987) investigate the effects of arrangements such as frequent-flyer programs in a two-period, homogeneous-product duopoly model. They show that such programs actually enable firms to achieve price collusion. Caminal and Matutes (1990) analyze the effect of alternative pricing policies on the competitive outcome in a two-period, differentiated-product duopoly model. They find that equilibrium profits will fall if firms can precommit to a second-period price for their loyal customers but will increase if the precommitment is for a discount to the loyal customers. In both of these papers, firms discriminate against newcomers in the second period and thus generate artificial switching costs for consumers. In my model, on the other hand, there are real costs from switching, but firms can make arrangements that artificially reduce the switching costs borne by consumers (using PCTS). Another important difference is that in these two papers a firm needs to be able to precommit to some future price or discount for its current customers, while here such commitment ability is not required.

My analysis is also related to studies on third-degree price discrimination, such as Homes (1989), but in that literature there is no specific analysis of consumer switching costs and there is no intertemporal consideration of equilibrium effects of price discrimination. Nilsen (1992) has considered consumer switching costs in a market with third-degree price discrimination, but he is concerned with how the market outcomes are affected by two different types of consumer switching costs, namely the transaction costs and the learning costs, and he focuses on a stationary equilibrium in a model of infinite horizons. Like others in the literature about consumer switching costs, Nilsen...
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sen's paper also has the feature that in equilibrium no consumer actually switches.³

2. Competition in the Mature Market: The Second Period

I start the analysis by looking at the second period (the mature market), where each firm has already established a market share, or customer base. In addition to being the basis of determining the outcomes in the first period, the equilibrium in the mature market is itself of interest, since in some situations one might want to consider the use of PCTS as a new form of marketing that is invented only in the mature market and is thus unanticipated when the market is new.

2.1 Equilibrium under PCTS

I first establish the following notation, for i = A, B:

\[ p_{i2} = \text{firm i's second-period price under PCTS}, \]

\[ m_i = \text{firm i's discount payment to a new customer switching from another firm}, \]

\[ q_{ij} = \text{number of consumers who bought from firm j previously but buy from firm i in the second period}. \]

A consumer belonging to firm A's market share, \( \alpha \), in the beginning of the second period is indifferent between continuing to buy from A and switching to buying from B if her s is such that⁴

\[ R - p_{A2} = R - p_{B2} - s + m_B. \]

Therefore,

\[ q_{AA} = \alpha \int_{p_{A2} - p_{B2} + m_B}^{\theta} \frac{1}{\theta} ds = \alpha \left( 1 - \frac{p_{A2} - p_{B2} + m_B}{\theta} \right); \]

and

3. An exception is Chen and Rosenthal (1996), which studies a dynamic duopoly model with consumer loyalties where there is equilibrium switching by consumers. However, in the equilibrium of their model firms use mixed strategies. A model with equilibrium consumer switching seems to be a more realistic description of the competition between rival firms in markets with repeated purchases. A recent TV advertisement by AT&T, for example, claimed that consumers switched from MCI to AT&T 8 million times in 1994.

4. Our assumption that \( R > c + \theta \) ensures that each consumer will indeed purchase from one of the firms in equilibrium. This is always true in the rest of the paper.
\[ q_{BA} = \frac{\alpha}{\theta} (p_{A2} - p_{B2} + m_B), \]

provided that
\[ 0 \leq p_{A2} - p_{B2} + m_B \leq \theta. \]

Similarly,
\[ q_{AB} = \frac{1 - \alpha}{\theta} (p_{B2} - p_{A2} + m_A); \]

and
\[ q_{BB} = (1 - \alpha) \left( 1 - \frac{p_{B2} - p_{A2} + m_A}{\theta} \right), \]

provided that
\[ 0 \leq p_{B2} - p_{A2} + m_A \leq \theta. \]

Firm A and firm B's profits in this period are, respectively,
\[ \pi_{A2} = \alpha (p_{A2} - c) \left( 1 - \frac{p_{A2} - p_{B2} + m_B}{\theta} \right) \]
\[ + \frac{1 - \alpha}{\theta} (p_{A2} - c - m_A)(p_{B2} - p_{A2} + m_A), \]
\[ \pi_{B2} = (1 - \alpha) (p_{B2} - c) \left( 1 - \frac{p_{B2} - p_{A2} + m_A}{\theta} \right) \]
\[ + \frac{\alpha}{\theta} (p_{B2} - c - m_B)(p_{A2} - p_{B2} + m_B). \]

A Nash equilibrium of this second-period stage game is a pair \((p_{A2}^{*}, m_A^{*})\) and \((p_{B2}^{*}, m_B^{*})\) (only pure strategies will be considered in this paper) such that \((p_{A2}^{*}, m_A^{*})\) maximizes \(\pi_{A2}\) given \((p_{B2}^{*}, m_B^{*})\), and \((p_{B2}^{*}, m_B^{*})\) maximizes \(\pi_{B2}\) given \((p_{A2}^{*}, m_A^{*})\). The first-order conditions are
\[ \frac{\partial \pi_{A2}}{\partial p_{A2}} = \alpha \left( 1 - \frac{p_{A2} - p_{B2} + m_B}{\theta} \right) - \frac{\alpha}{\theta} (p_{A2} - c) \]
\[ + \frac{1 - \alpha}{\theta} (p_{B2} - p_{A2} + m_A) - \frac{1 - \alpha}{\theta} (p_{A2} - c - m_A) = 0, \]
\[ \frac{\partial \pi_{A2}}{\partial m_A} = \frac{1 - \alpha}{\theta} (p_{A2} - c - m_A) - \frac{1 - \alpha}{\theta} (p_{B2} - p_{A2} + m_A) = 0, \]
\[
\frac{\partial \pi_{B2}}{\partial p_{B2}} = (1 - \alpha) \left(1 - \frac{p_{B2} - p_{A2} + m_A}{\theta}\right) - \frac{1 - \alpha}{\theta} (p_{B2} - c)
\]
\[
+ \frac{\alpha}{\theta} (p_{A2} - p_{B2} + m_B) - \frac{\alpha}{\theta} (p_{B2} - c - m_B) = 0,
\]
\[
\frac{\partial \pi_{B2}}{\partial m_B} = \frac{\alpha}{\theta} (p_{B2} - c - m_B) - \frac{\alpha}{\theta} (p_{A2} - p_{B2} + m_B) = 0.
\]

The system of equations above has the following unique solution:
\[
p_{A2}^* = p_{B2}^* = c + \frac{3}{2} \theta, \quad m_A^* = m_B^* = \frac{3}{2} \theta,
\]
and the second-order conditions are satisfied. We therefore have:

**Proposition 1**: There exists a unique Nash equilibrium in the mature market under PCTS. In this equilibrium, each firm's price is independent of its market share, and is always higher than marginal cost but below the monopoly price \( R \).

The lack of dependence of competing firms' prices on their respective market shares under PCTS is somewhat surprising, since in the presence of consumer switching costs one usually expects that the firm with higher market shares will price higher to exploit the locked-in customers—which is also true in the present model under UNIF, as we shall see shortly. In other papers in the switching-costs literature where no price discrimination is allowed, such as in Klemperer (1987b), the market-share independence of a firm's equilibrium prices can hold only if consumer switching costs are high enough so that each firm is able to charge the monopoly price and no consumer actually switches. In our case here, both firms' prices are lower than \( R \), the monopoly price. Moreover, substituting result (1) into the equations for \( q_{AB} \) and \( q_{BA} \), we obtain in equilibrium \( q_{AB} = (1 - \alpha)/3 \) and \( q_{BA} = \alpha/3 \). Thus exactly \( \frac{1}{3} \) of the consumer population switch suppliers here.\(^5\)

From result (1), one also notices that the new-customer discounts offered by each firm increase continuously in \( \theta \), but the prices for non-switching customers and the after-discount prices for switching customers are also both increasing in \( \theta \). Since the expected switching cost of a typical consumer is \( \theta/2 \), which is higher as \( \theta \) becomes higher, and since consumer switching costs approach zero when \( \theta \) approaches zero, we have:

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5. This implies that a firm with a higher market share in the first period will have a reduction of its market share in the second period. One may wonder why the firm would not cut its price to stop the attrition of its market share. The reason is that cutting prices would lower the firm’s profit on those consumers who do buy from the firm. The equilibrium balances the two different effects of cutting prices.
Proposition 2: As the expected switching cost of a typical consumer increases, the actual prices paid by all consumers increase continuously, and so does the difference of the actual prices paid by switching and non-switching customers. Furthermore, when switching costs approach zero, the equilibrium market prices approach the marginal cost and the new-customer discount payments approach zero.

It has been shown in the literature that the presence of consumer switching costs generally makes the market less competitive (Klepper, 1995). The analysis here has confirmed this result, even though in my model firms can pay consumers to switch. My analysis also suggests that the presence of consumer switching costs alone would be able to explain why PCTS occurs as a commonly used pricing strategy. Moreover, as switching costs increase, there will be more price discrimination in the market.

Substituting the equilibrium prices and discount payments into the second-period profit functions, we obtain

\[ \pi_{A2} = \frac{\theta}{3} \left( \frac{1}{3} + \alpha \right), \quad \pi_{B2} = \frac{\theta}{3} \left( \frac{1}{3} - \alpha \right). \quad (2) \]

Although in equilibrium both firms charge the same prices for a homogeneous product, one-third of the consumer population change suppliers in the mature market under PCTS. From a society's point of view, the switching costs involved are clearly a dead-weight loss. Since any consumer with \( s < \theta/3 \) switches in equilibrium, the expected dead-weight loss per consumer is \( \int_0^{\theta/3} s(1/\theta) \, ds = \theta/18 \).

2.2 Comparison with the equilibrium under unif

Under unif, let \( p_{i2} \) be firm \( i \)'s price in the second period, \( q_i \) be the number of consumers who would buy from firm \( i \) in the second period, and \( \pi_{i2} \) be firm \( i \)'s second-period profits, where \( i = A, B \). Suppose first that \( p_{A2} \geq p_{B2} \).

A consumer belonging to \( \alpha \) is indifferent between continuing buying from \( A \) and switching to buying from \( B \) if her \( s \) is such that

\[ R - p_{A2} = R - p_{B2} - s. \]

Therefore,

\[ q_{i2} = \alpha \int_{p_{A2} - p_{B2}}^{\theta} \frac{1}{\theta} \, ds = \alpha \left( 1 - \frac{p_{A2} - p_{B2}}{\theta} \right) \]

and
\[ q_B^* = \frac{\alpha}{\theta} (p_A^* - p_B^*) + (1 - \alpha). \]

Firm A and firm B’s profits in the mature market under UNIF are, respectively:

\[ \pi_A^* = \alpha (p_A^* - c) \left( 1 - \frac{p_A^* - p_B^*}{\theta} \right), \]
\[ \pi_B^* = (p_B^* - c) \left( \frac{\alpha}{\theta} (p_A^* - p_B^*) + (1 - \alpha) \right). \]

The first-order conditions are

\[ \frac{\partial \pi_A^*}{\partial p_A^*} = \alpha \left( 1 - \frac{p_A^* - p_B^*}{\theta} \right) - \frac{\alpha}{\theta} (p_A^* - c) = 0, \]
\[ \frac{\partial \pi_B^*}{\partial p_B^*} = \frac{\alpha}{\theta} (p_A^* - p_B^*) + (1 - \alpha) - \frac{\alpha}{\theta} (p_B^* - c) = 0. \]

There is a unique pair \((p_A^*, p_B^*)\) that solves the two equations above and satisfies the second-order conditions, where

\[ p_A^* = c + \frac{1 + \alpha}{3\alpha} \theta, \quad p_B^* = c + \frac{2 - \alpha}{3\alpha} \theta. \]

Clearly, \(p_A^* \geq p_B^*\) if and only if \(\alpha \geq \frac{1}{2}\).

Similarly, if \(p_A^* \leq p_B^*\), one obtains

\[ p_A^* = c + \frac{1 + \alpha}{3(1 - \alpha)} \theta, \quad p_B^* = c + \frac{2 - \alpha}{3(1 - \alpha)} \theta, \]

and \(p_A^* \leq p_B^*\) if and only if \(\alpha \leq \frac{1}{2}\). Therefore, under UNIF, there is a unique Nash equilibrium in the second period of the game, where the two firms’ equilibrium strategies are

\[ p_A^* = \begin{cases} c + \frac{1 + \alpha}{3\alpha} \theta & \text{if } \alpha \geq \frac{1}{2}, \\ c + \frac{1 + \alpha}{3(1 - \alpha)} \theta & \text{if } \alpha < \frac{1}{2}. \end{cases} \tag{3} \]

\[ p_B^* = \begin{cases} c + \frac{2 - \alpha}{3\alpha} \theta & \text{if } \alpha \geq \frac{1}{2}, \\ c + \frac{2 - \alpha}{3(1 - \alpha)} \theta & \text{if } \alpha < \frac{1}{2}. \end{cases} \tag{4} \]
In contrast to PCTS, a firm with higher market shares charges higher prices under UNIF. Also notice that both firms’ prices are at the highest when they have the equal market shares, namely \( \alpha = \frac{1}{2} \), where \( p_A^* = p_B^* = c + \theta \). If \( \alpha \geq \frac{1}{2} \), both firms’ prices are at the lowest when \( \alpha = 1 \), where \( p_A^* = c + \frac{2}{3} \theta \) and \( p_B^* = c + \frac{1}{3} \theta \). If \( \alpha \leq \frac{1}{2} \), both firms’ prices are at the lowest when \( \alpha = 0 \), where \( p_A^* = c + \frac{1}{3} \theta \) and \( p_B^* = c + \frac{2}{3} \theta \). Recall that under PCTS, both firms’ customers who do not switch pay \( c + \frac{2}{3} \theta \), and those who switch effectively pay \( c + \frac{1}{3} \theta \) after receiving the discount, independent of the market-share distributions. In addition, a switching customer under PCTS incurs a switching cost that is no higher than \( \frac{1}{3} \theta \). We therefore have:

**Proposition 3:** In the mature market, all consumers will have higher surpluses in equilibrium under PCTS than under UNIF if the two firms have equal market shares.

If two firms have different market shares, then consumers who previously bought from the firm with the higher market share will pay a lower price in the second period under PCTS than under UNIF, but a consumer who previously bought from the firm with the lower market share may pay a higher price under PCTS than under UNIF if her switching cost is higher than \( \frac{1}{3} \) and the other firm’s market share is sufficiently close to 1.

Firm A and firm B’s equilibrium profits in the mature market under UNIF are

\[
\pi_A^* = \begin{cases} 
(1 + \alpha)^2 \theta / 9 \alpha & \text{if } \alpha \geq \frac{1}{2}, \\
(1 + \alpha)^2 \theta / 9(1 - \alpha) & \text{if } \alpha < \frac{1}{2},
\end{cases}
\]

(5)

\[
\pi_B^* = \begin{cases} 
(2 - \alpha)^2 \theta / 9 \alpha & \text{if } \alpha \geq \frac{1}{2}, \\
(2 - \alpha)^2 \theta / 9(1 - \alpha) & \text{if } \alpha < \frac{1}{2}.
\end{cases}
\]

(6)

It is easy to see that firm A’s profit will be higher than firm B’s if and only if A has a higher market share than B. However, each firm’s profit is maximized at \( \alpha = \frac{1}{2} \), where \( \pi_A^* = \pi_B^* = \theta / 2 \). If \( \alpha \geq \frac{1}{2} \), each firm’s profit decreases in \( \alpha \) and is minimized at \( \alpha = 1 \), where \( \pi_A^* = \frac{4}{3} \theta \) and \( \pi_B^* = \frac{2}{3} \theta \). If \( \alpha < \frac{1}{2} \), each firm’s profit increases in \( \alpha \) and is minimized at \( \alpha = 0 \), where \( \pi_A^* = \frac{1}{3} \theta \) and \( \pi_B^* = \frac{2}{3} \theta \). Note also that \( \pi_A^* \) and \( \pi_B^* \)
are not differentiable with respect to $\alpha$ at $\alpha = \frac{1}{2}$. Comparing (5) and (6) with (2), since

$$\frac{\theta}{3} \left( \frac{1}{3} + \alpha \right) - \frac{(1 + \alpha)^2}{9\alpha} \theta = -\frac{(1 - \alpha)(2\alpha + 1)}{9\alpha} \theta,$$

$$\frac{\theta}{3} \left( \frac{1}{3} + \alpha \right) - \frac{(1 + \alpha)^2}{9(1 - \alpha)} \theta = -\frac{4\alpha^2}{9(1 - \alpha)} \theta;$$

$$\frac{\theta}{3} \left( \frac{1}{3} - \alpha \right) - \frac{(2 - \alpha)^2}{9\alpha} \theta = -\frac{4(1 - \alpha)^2}{9\alpha} \theta,$$

$$\frac{\theta}{3} \left( \frac{1}{3} - \alpha \right) - \frac{(2 - \alpha)^2}{9(1 - \alpha)} \theta = -\frac{\alpha(3 - 2\alpha)}{9(1 - \alpha)} \theta,$$

we have:

**Proposition 4:** In the mature market, both firms' equilibrium profits are lower under *pcts* than under *unif*.

In the equilibrium under *unif*, the portion of switching customers is $(\alpha/\theta)(p_{A2}^* - p_{B2}^*) = (2\alpha - 1)/3$ if $\alpha \geq \frac{1}{2}$, and $[(1 - \alpha)/\theta](p_{A2}^* - p_{B2}^*) = (1 - 2\alpha)/3$ if $\alpha < \frac{1}{2}$. Since under *pcts* the portion of switching customers is always $\frac{1}{3}$, there are more consumer switching and higher expected aggregate switching costs under *pcts*. Therefore:

**Proposition 5:** In the equilibrium of the mature market, the dead-weight loss to society due to consumer switching is higher under *pcts* than under *unif*.

Notice that the equilibrium outcomes in the market under both *pcts* and *unif* vary continuously in $\theta$. When $\theta$ approaches zero (switching costs approach zero), prices, profits, and consumer surplus under both regimes approach those in a market without switching costs.

### 3. Competition in the New Market: The First Period

We now examine the subgame-perfect equilibrium outcomes in the first period where consumers are not attached to any firm.

#### 3.1 Equilibrium under *pcts*

Under *pcts*, both firms charge the same prices in the second period. Therefore, in the first period all consumers will purchase from firm $i$ if $i$’s price is lower than $j$’s, where $i \neq j$, $i, j \in \{A, B\}$. When both firms have the same prices, I assume that a consumer buys from either firm
with equal probability. Let \( q_{i1} \) be the quantity and \( p_{i1} \) the price for firm \( i \) in the first period under PCTS.

**Proposition 6:** Under PCTS, there exists a unique subgame-perfect equilibrium of the model where in the first period each firm’s price is \( p_{A1}^* = p_{B1}^* = c - (\delta/3)\theta \), and in the second period, each firm’s strategies are given by result (1).

**Proof.** At the proposed equilibrium, the discounted sum of expected profits of each firm (of the two periods together) is

\[
\frac{1}{2} \left( c - \frac{\delta}{3} \theta - c \right) + \delta \theta \left( \frac{1}{9} + \frac{1}{6} \right) = \frac{\delta}{9} \theta.
\]

If a firm deviates to any price lower than \( c - \delta/3 \), it will sell to all customers in the first period and the discounted sum of its expected profit will be less than

\[
c - \frac{\delta}{3} - c + \delta \theta \left( \frac{1}{9} + \frac{1}{3} \right) = \frac{\delta}{9} \theta.
\]

If a firm deviates to any price higher than \( c - \delta/3 \), it sells zero in the first period and the discounted sum of its expected profit will be its discounted expected profit in the second period, \((\delta/9)\theta\). Thus, the proposed strategies indeed constitute a subgame-perfect equilibrium.

To show the uniqueness of the subgame-perfect equilibrium, suppose that there is another equilibrium pair of prices in the first period, \((p_{A1}, p_{B1})\). Without loss of generality, suppose \( p_{A1} \leq p_{B1} \). If \( p_{B1} = c - (\delta/3)\theta \) and \( p_{A1} < p_{B1} \), or if \( p_{B1} < c - (\delta/3)\theta \), then the discounted sum of firm A’s expected profit will be less than \((\delta/9)\theta\), and therefore firm A will benefit from deviating to a price \( c - (\delta/9)\theta \) or higher. If \( p_{B1} > c - (\delta/3)\theta \), then in order for \( p_{A1} \) to be optimal for firm A, one must have \( p_{B1} \geq p_{A1} > c - (\delta/3)\theta \). If \( p_{A1} < p_{B1} \), firm B’s expected profit will be \((\delta/9)\theta\); and if \( p_{A1} = p_{B1} \), firm B’s expected profit will be

\[
\frac{1}{2} (p_{B1} - c) + \frac{5\delta}{18} \theta.
\]

In either case, firm B’s price cannot be optimal given \( p_{A1} \), since it can lower its price to below \( p_{A1} \) but still above \( c - \delta/3 \), yielding a profit of

\[
p_{B1} - c + \frac{4\delta}{9} \theta,
\]

which is above \((\delta/9)\theta\) and is also above \( \frac{1}{2}(p_{B1} - c) + (5\delta/18)\theta \) when \( p_{B1} > c - (\delta/3)\theta \). \(\square\)
Thus, under PCTS, the competition for market share leads to a price below marginal cost for both firms in the first period. The discounted sum of expected equilibrium profits of each firm is \((\delta/9)\theta\). The discounted sum of expected consumer surpluses is

\[
R - c + \frac{\delta}{3} \theta + \delta \left[ R - \frac{2}{3} \left( c + \frac{2}{3} \theta \right) - \frac{1}{3} \left( c + \frac{1}{3} \theta + \frac{1}{6} \theta \right) \right]
\]

\[
= (1 + \delta)(R - c) - \frac{5\delta}{18} \theta.
\]

Notice that switching costs raise the discounted sum of each firm’s profits and reduce the discounted sum of consumer surpluses. As \(\theta\) goes to zero, the equilibrium approaches the equilibrium of an otherwise identical market without switching costs.

### 3.2 Comparison with the Equilibrium under UNIF

Under UNIF, a firm with a higher market share will charge a higher price in the second period. In deciding from which firm to buy in the first period, rational consumers should take this into account. Denote each firm’s first-period price under UNIF by \(p^{u}_{i1}, i = A, B\). A subgame-perfect equilibrium is a pair of prices \((p^{u}_{A1}, p^{u}_{B1})\) and a partition of the consumer population in the first period into those buying from firm \(A\) \((\alpha)\) and those buying from firm \(B\) \((1 - \alpha)\), together with the two firms’ second-period equilibrium strategies such that each consumer and each firm is optimizing given the strategies of all other players.

Suppose first that \(\alpha \geq \frac{1}{2}\), which implies that \(p^{u}_{A2} \geq p^{u}_{B2}\). Any consumer is indifferent between buying from firm \(A\) and buying from firm \(B\) in the first period if she expects the same discounted sum of total surpluses from either firm, that is, if

\[
R - p^{u}_{A1} + \delta \left( R - \int_{p^{u}_{A2} - p^{u}_{B2}}^{\theta} p^{u}_{A2} \frac{1}{\theta} ds - \int_{0}^{p^{u}_{A2} - p^{u}_{B2}} (p^{u}_{B2} + s) \frac{1}{\theta} ds \right)
\]

\[
= R - p^{u}_{B1} + \delta (R - p^{u}_{B2}),
\]

which can be simplified to

\[
p^{u}_{A1} - p^{u}_{B1} + \delta \left( -\frac{1}{2\theta} (p^{u}_{A2} - p^{u}_{B2})^2 + (p^{u}_{A2} - p^{u}_{B2}) \right) = 0.
\]

From (3) and (4), we have \(p^{u}_{A2} - p^{u}_{B2} = (\theta/3)(2\alpha - 1)/\alpha\) for \(\alpha \geq \frac{1}{2}\). Substituting this into the above equation, we obtain the relation between \(\alpha\) and both firms’ first-period prices:
\[ p_A^* - p_B^* + \frac{\delta \theta (2\alpha - 1)(4\alpha + 1)}{18\alpha^2} = 0. \quad (7) \]

Thus, given each firm’s price and the requirement of subgame perfection, each consumer's choice is optimal if and only if the above equation holds. Notice that \( \alpha = \frac{1}{2} \) if \( p_A^* = p_B^* \). Differentiating \( \alpha \) with respect to \( p_A^* \) and \( p_B^* \), using the implicit differentiation rule, we obtain

\[ \frac{\partial \alpha}{\partial p_A^*} = -\frac{\partial \alpha}{\partial p_B^*} = -\frac{9\alpha^3}{\delta \theta (1 + \alpha)}. \]

Next, if \( \alpha < \frac{1}{2} \), which implies that \( p_A^* < p_B^* \), a similar derivation results in

\[ p_B^* - p_A^* + \frac{\delta \theta (1 - 2\alpha)(5 - 4\alpha)}{18(1 - \alpha)^2} = 0. \quad (8) \]

Also,

\[ \frac{\partial \alpha}{\partial p_A^*} = -\frac{\partial \alpha}{\partial p_B^*} = -\frac{9(1 - \alpha)^3}{\delta \theta (2 - \alpha)}. \]

Thus both \( \partial \alpha/\partial p_A^* \) and \( \partial \alpha/\partial p_B^* \) exist at \( \alpha = \frac{1}{2} \), and hence they exist for \( 0 \leq \alpha \leq 1 \).

Now, the first-period profits of the two firms are

\[ \pi_A^* = (p_A^* - c)\alpha, \quad \pi_B^* = (p_B^* - c)(1 - \alpha). \]

The discounted sums of each firm's profits are

\[ \pi_A = (p_A^* - c)\alpha + \delta \pi_A^*, \quad \pi_B = (p_B^* - c)(1 - \alpha) + \delta \pi_B^*. \]

The intriguing issue here is that since each firm's second-period profit is not differentiable with respect to \( \alpha \) at \( \alpha = \frac{1}{2} \) (\( \pi_A^* \) and \( \pi_B^* \) each have different left and right derivatives at \( \alpha = \frac{1}{2} \)), we may not be able to apply the usual first-order conditions to \( \pi_A^* \) and \( \pi_B^* \). However, since both \( \pi_A^* \) and \( \pi_B^* \) are uniquely maximized at \( \alpha = \frac{1}{2} \), if we can find some \( p_A^* = p_B^* \) that constitutes a Nash equilibrium in the first-period one-shot game (notice that \( p_A^* = p_B^* \) implies \( \alpha = \frac{1}{2} \)) where each firm’s payoff is its first-period profit only, then this pair of first-period prices must induce a subgame-perfect equilibrium in the entire game.

**Proposition 7:** Under unif, the full model has a subgame-perfect Nash equilibrium where each firm’s first-period price equals \( c + \frac{\delta \theta}{6} \), and the second-period strategies are given by equations (3) and (4), with \( \pi_A^* = \pi_B^* = \frac{5}{6} \delta \theta \).

**Proof.** For \( \alpha \geq \frac{1}{2} \), the first-order conditions in the first-period one-shot game are
\[
\frac{\partial \pi^A_1}{\partial p^A_1} = \alpha - (p^A_1 - c) \frac{9\alpha^3}{\delta\theta(1 + \alpha)} = 0,
\]
\[
\frac{\partial \pi^B_1}{\partial p^B_1} = 1 - \alpha - (p^B_1 - c) \frac{9\alpha^3}{\delta\theta(1 + \alpha)} = 0.
\]

The unique solution to the two equations above and equation (7) is
\[p^*_A = p^*_B = c + \frac{2}{3}\delta\theta\] and \[\alpha = \frac{1}{2},\] and the second-order conditions are satisfied. We can also find \[\frac{\partial \pi^A_1}{\partial p^A_1} \text{ and } \frac{\partial \pi^B_1}{\partial p^B_1}\] when \[\alpha < \frac{1}{2},\] and one can verify that both \[\frac{\partial \pi^A_1}{\partial p^A_1} \text{ and } \frac{\partial \pi^B_1}{\partial p^B_1}\] are continuous for all \[\alpha \in [0, 1],\] and there exists no solution to the equations \[\frac{\partial \pi^A_1}{\partial p^A_1} = 0, \frac{\partial \pi^B_1}{\partial p^B_1} = 0,\] and equation (8) when \[\alpha < \frac{1}{2}.\] Thus \[p^*_A = p^*_B = c + \frac{2}{3}\delta\theta\] constitutes the unique Nash equilibrium in the first-period one-shot game, and induces a subgame-perfect Nash equilibrium of the game where the firms' second-period strategies are given by equations (3) and (4). Each firm's discounted sum of profits at this equilibrium is \[\frac{1}{2} \cdot \frac{2}{3}\delta\theta + \frac{1}{2}\delta\theta = \frac{5}{6}\delta\theta.\]

Because a firm with a higher market share will charge a higher price under UNIF, the consumer demand facing each firm in the first period becomes less elastic under UNIF than under PCTS. This in turn can support higher equilibrium first-period prices under UNIF. Interestingly, because both \[\pi^A_2\] and \[\pi^B_2\] are maximized at \[\alpha = \frac{1}{2}\] but each has different left and right derivatives at \[\alpha = \frac{1}{2},\] there can be symmetric subgame-perfect Nash equilibria of the full model for a range of first-period prices. For instance, \[p^*_A = p^*_B = c - \frac{27}{36}\delta\theta\] together with \[p^*_A\] and \[p^*_B\] given by equations (3) and (4) constitutes another subgame-perfect Nash equilibrium of the model under UNIF, with \[\pi^*_A = \pi^*_B = \frac{1}{2}\delta\theta.\] To see this, first notice that \(A\) cannot benefit by unilaterally deviating to any price lower than \(c - \frac{27}{36}\delta\theta\) in the first period. Next, if \[p^*_A > p^*_B = c - \frac{27}{36}\delta\theta,\] then \(\alpha < \frac{1}{2}\) and \[\pi^*_A = (p^*_A - c)\alpha + \delta(1 + \alpha/2)[9(1 - \alpha)] \theta.\] One can verify that, given \[p^*_B = c - \frac{27}{36}\delta\theta,\] \[\pi^*_A\] decreases in \(\alpha\) for \(\alpha \in (0, \alpha_1)\) and increases in \(\alpha\) for \(\alpha \in (\alpha_1, \frac{1}{2})\), where \(\alpha_1 \approx 0.23544.\) But since \[\pi^*_A = \frac{1}{2}\delta\theta\] when \(\alpha = 0,\) firm \(A\) cannot benefit from unilaterally deviating to any price higher than \(c - \frac{27}{36}\delta\theta\) in the first period. A similar argument establishes that \(B\) cannot benefit from unilaterally deviating to any price different from \(c - \frac{27}{36}\delta\theta\) at the first period.

Thus, it is possible that in equilibrium firms charge lower prices in the first period under UNIF than under PCTS. However, we have

**Proposition 8:** In subgame-perfect equilibrium, the discounted sum of each firm's profits is weakly higher under UNIF than under PCTS.

**Proof.** In equilibrium, the discounted sum of each firm's profits under PCTS is \(\delta\theta/9,\) from Proposition 6. From Proposition 7, it is clear that
there exists a subgame-perfect equilibrium under \textit{UNIF} with a higher discounted sum of profits for each firm. We now show that for any subgame-perfect Nash equilibrium under \textit{UNIF}, $\pi_i^{**} \geq \delta \theta / 9$ for $i = A, B$. But this is obvious, since by charging $p_{i1}^* = c$, firm $i$ will have zero first-period profit for any $p_{j1}, j \neq i$, and will have a discounted second-period profit of at least $\delta \theta / 9$.

Therefore, firms are worse off under \textit{PCTS}. However, consumers may or may not be better off under \textit{PCTS}. If under \textit{UNIF} $p_{A1}^{\text{UNIF}} = p_{B1}^{\text{UNIF}} = c + \frac{3}{2} \delta \theta$, for instance, then consumers are surely better off under \textit{PCTS}. On the other hand, if in equilibrium $p_{A1}^{\text{PCTS}} = p_{B1}^{\text{PCTS}} = c - \frac{27}{30} \delta \theta$, then a consumer's discounted sum of expected surplus would be $(1 + \delta)(R - C) - \frac{9}{30} \delta \theta$ under \textit{UNIF}, compared to $(1 + \delta)(R - C) - \frac{10}{30} \delta \theta$ under \textit{PCTS}. Thus, firms and consumers can both be worse off in equilibrium under \textit{PCTS} than under \textit{UNIF}. This is because under \textit{PCTS} there are dead-weight losses associated with consumer switching, while there are no such losses at any symmetric equilibrium under \textit{UNIF}.

Recall from Proposition 5 that in equilibrium there are always higher switching costs incurred under \textit{PCTS}. Since the output is the same under \textit{PCTS} and under \textit{UNIF}, the difference in social welfares between these two price regimes is determined by the difference in switching costs actually incurred. Therefore, social welfare is clearly lower under \textit{PCTS} than under \textit{UNIF}.

Although the pricing strategies and the environment in this paper are rather different from those in Caminal and Matutes (1990), the results in the two papers share some common features. For instance, having the choice to engage in price discrimination actually makes both firms worse off in our model, and the same can be true in Caminal and Matutes. Also, there are dead-weight losses in the equilibria of both models due to more expenses in transportation cost (in Caminal and Matutes) and switching cost (in the present paper). Important differences do exist between the outcomes of the two models, however. In particular, market prices in Caminal and Matutes decrease over time, while prices under \textit{PCTS} in our model increase over time. Furthermore, in a mature market each firm's price, if it has not been precommitted in the earlier period, is increasing in its market share in Caminal and Matutes, while each firm's price is independent of its market share in our model under \textit{PCTS}.

6. There is an important qualification for our welfare result: output is not changed by the use of \textit{PCTS} in this model. If output increased under \textit{PCTS} due to more competitive prices, then the effects of such changes would need be taken into account in our welfare comparisons. There is more on this issue in the next section.
4. Discussion

This paper has studied the practice of competing firms paying customers to switch in a market with switching costs. The use of PCTS leads to equal prices by firms in a mature market even if they have different market shares, and thus to more elastic consumer demand in a new market. The presence of switching costs alone would be able to explain the use of PCTS, and higher switching costs result in more price discrimination in equilibrium. A mature market is more competitive under PCTS than in the case where price discrimination is not feasible, but even under PCTS prices increase with the expected size of switching costs. Under PCTS, firms are worse off, but consumers need not always be better off. The equilibrium switching of consumers creates a higher dead-weight loss under PCTS.

My analysis is based on a stylized model. This raises the question of to what extent the results are robust. In particular, the result that prices become independent of market shares under PCTS (to be called the independence result in what follows) appears unusual, and one might wonder whether it will continue to hold in a more general model. One way to extend the model, for instance, is to replace the uniform distribution with more general distributions of switching costs. We have the following:

**Proposition 9:** Suppose the model described in Section 1 is extended so that consumer switching cost is any continuous random variable distributed on \([0, 5]\), where \(0 < 5 \leq \infty\). Also assume that \(R\) is sufficiently high and the second-period profit of each firm is a concave function of its own strategic variables, so that equilibrium exists in the second-period stage game. Then under PCTS, firms' equilibrium prices in the second period are independent of their respective market shares.

**Proof.** See Appendix.

To extend the model in other directions may prove to be more difficult. For instance, it would be interesting to allow the possibility that there are new uncommitted consumers entering the market in period 2 and firms cannot distinguish them from rivals' customers. Extending the model in that direction, however, runs into the difficulty of nonexistence of pure-strategy equilibrium, since profits will no longer be continuous in prices.7 This difficulty may be avoided if a

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7. When the two firms charge the newcomers the same (effective) prices, a firm can attract all the newcomers by reducing its price slightly and thus increase its profit discontinuously.
different model is used so that the two firms' products are differentiated to consumers *ex ante*, but that is beyond the scope of this paper. It is still desirable, however, to extend our analysis in such a direction in future research.

While one should be cautious in interpreting the independence result beyond the context of my model, I am nevertheless convinced that the intuition behind this result can provide valid insights in a much more general setting. In the case of firms charging the same prices for all its customers, a firm faces the following trade-off. On the one hand, by raising its price the firm can earn higher profits on its partially locked-in customers. On the other hand, by lowering its price, it is able to attract more new customers from its rival firm. The higher is the firm’s market share, the more important is the first effect and thus the higher is the firm’s price. Under *PCTS*, the trade-off is no longer affected by market shares, and thus neither are prices, since the firm is able to charge different prices for its partially locked-in customers and for the customers switching from its rival.

I now turn to the welfare implications of the results. In my model, social welfare is lower under *PCTS*. This result, however, need not hold in a more general model where an individual consumer's demand is elastic. The use of *PCTS* would then potentially have two opposite effects on social welfare. On the negative side is the dead-weight loss associated with consumer switching that has been identified in this paper. On the positive side, the market may become more competitive under *PCTS*, which will reduce the dead-weight loss caused by output distortion when prices exceed marginal cost. Which of these two effects dominates is likely to depend on, among other things, how elastic consumer demand is. This suggests that a desirable public policy toward the business practice of *PCTS* will likely depend on the particular industries that are involved, and that studies with industry-specific demand conditions will be needed before any such policy can be formed.

Another important assumption in my model is that a consumer learns her switching cost at the beginning of the second period, although she knows the possible distribution of this cost in the very beginning. An alternative assumption would be that consumers know their future switching costs before they make the first purchase. It does

8. Indeed, in assuming in this paper that firms produce a homogeneous product, my aim is to focus on the effects of consumer switching costs. If firms produce differentiated products, then consumers may be unwilling to change suppliers even if there are no switching costs.

9. I thank a referee for suggesting this intuitive explanation of the independence result.
not seem obvious to me that this alternative assumption would be more compelling, however. What I have in mind are situations where, for example, at the time a customer opens an account with a bank, signs on with a long-distance telephone company, or makes the first purchase of a particular line of software, there are many unknown factors that may affect her cost to switch to another bank, another company, or another line of software later on; she has some expectations about what these costs will be, but she can learn them only when it is time for her to decide whether or not to switch. Another reason why I have not chosen this alternative assumption is that the model would then become much more difficult to handle. This is because the distribution of switching costs at the beginning of the second period would then depend on the strategies of the consumers and the firms in the first period, and the concept of subgame-perfect equilibrium would in that case lose its power.

There are several other directions in which to extend our analysis in future research. For instance, there could be more than two firms in the market; or firms might enter the market at different points of time.\textsuperscript{10} Also, to understand the steady state of the market under PCTS, it would be interesting to consider a model with infinite horizons.\textsuperscript{11}

The existing literature has largely focused on the question of how equilibrium outcomes in a market with consumer switching costs differ from those in an otherwise identical market without switching costs. The present paper is a step toward answering the question of how market outcomes differ under different forms of price competition, given that there are real consumer switching costs in the market. In many situations, the latter seems to be a more relevant question from the perspectives of both business strategy and public policy, and more studies on it would be desirable.

**APPENDIX**

**Proof of Proposition 9.** Denote the p.d.f. and the c.d.f. of the switching costs by $f(s)$ and $F(s)$ respectively. For sufficiently high $R$, we then have $q_{AA} = \alpha [1 - F(p_{A2} - p_{B2} + m_B)]$.

\textsuperscript{10} For an interesting recent paper that studies competition in markets with switching costs that involves entry by a new firm, see Wang and Wen (1995).

\textsuperscript{11} The consideration of switching costs has also motivated models of consumer loyalties, such as Rosenthal (1982) and Wernerfelt (1991). My analysis can be easily adopted to markets where consumers have certain loyalties for reasons that may not be entirely rational.
\[ q_{BA} = \alpha F(p_{A2} - p_{B2} + m_B), \]
\[ q_{AB} = (1 - \alpha)F(p_{B2} - p_{A2} + m_A), \]
\[ q_{BB} = (1 - \alpha)[1 - F(p_{B2} - p_{A2} + m_A)]. \]

The second-period profits of the two firms are
\[ \pi_{A2} = (p_{A2} - c)q_{AA} + (p_{A2} - c - m_A)q_{AB}, \]
\[ \pi_{B2} = (p_{B2} - c)q_{BB} + (p_{B2} - c - m_B)q_{BA}. \]

The first-order conditions are
\[ \frac{\partial \pi_{A2}}{\partial p_{A2}} = q_{AA} - \alpha(p_{A2} - c)f(p_{A2} - p_{B2} + m_B) \]
\[ + q_{AB} - (1 - \alpha)(p_{A2} - c - m_A)f(p_{B2} - p_{A2} + m_A) = 0, \]
\[ \frac{\partial \pi_{A2}}{\partial m_A} = -q_{AB} + (1 - \alpha)(p_{A2} - c - m_A)f(p_{B2} - p_{A2} + m_A), \]
\[ \frac{\partial \pi_{B2}}{\partial p_{B2}} = q_{BB} - (1 - \alpha)(p_{B2} - c)f(p_{B2} - p_{A2} + m_A) \]
\[ + q_{BA} - \alpha(p_{B2} - c - m_B)f(p_{A2} - p_{B2} + m_B) = 0, \]
\[ \frac{\partial \pi_{B2}}{\partial m_B} = -q_{BA} + \alpha(p_{B2} - c - m_B)f(p_{A2} - p_{B2} + m_B) = 0. \]

These first-order conditions can be simplified to the following system of equations:
\[ 1 - F(p_{A2} - p_{B2} + m_B) - (p_{A2} - c)f(p_{A2} - p_{B2} + m_B) = 0, \]
\[ -F(p_{B2} - p_{A2} + m_A) + (p_{A2} - c - m_A)f(p_{B2} - p_{A2} + m_A) = 0, \]
\[ 1 - F(p_{B2} - p_{A2} + m_A) - (p_{B2} - c)f(p_{B2} - p_{A2} + m_A) = 0, \]
\[ -F(p_{A2} - p_{B2} + m_B) + (p_{B2} - c - m_B)f(p_{A2} - p_{B2} + m_B) = 0. \]

Since \( \alpha \) does not appear in any of the four equations above, the equilibrium values of \( p_{i2} \) and \( m_i \) for \( i = A, B \), which are the solutions to these equations, will not depend on either \( \alpha \) or \( 1 - \alpha \).

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**References**


